TRAFFIC ASSIGNMENT BY SYSTEMS ANALYSIS

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by

W. A. McLAUGHLIN

Joint Highway Research Project

PURDUE UNIVERSITY
LAFAYETTE INDIANA

Final Report

TRAFFIC ASSIGNMENT BY SYSTEMS ANALYSIS

To: K. B. Woods, Director
Joint Highway Research Project

May 18, 1965

From: H. L. Michael, Associate Director
Joint Highway Research Project

File: 3-3-34
Project: C-36-54HH

Attached is a Final Report entitled "Traffic Assignment by Systems Analysis" by Mr. W. A. McLaughlin, Graduate Assistant on our staff. The research conducted by Mr. McLaughlin which resulted in this report was directed by Professor W. L. Grecco of our staff and has also been conducted by Mr. McLaughlin for his Ph.D. thesis.

The research reported was approved for implementation by the Board on June 19, 1964. Mr. McLaughlin did a substantial part of the research in absentia with facilities of the University of Waterloo in Canada. Mr. McLaughlin currently is on the staff of the Department of Civil Engineering of the University of Waterloo where he will become Assistant Head of Department at the beginning of the next academic year.

The report is presented to the Board for the record.

Respectfully submitted,

H. L. Michael BC

Harold L. Michael, Secretary

HLM:bc

Attachment

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Final Report

TRAFFIC ASSIGNMENT BY SYSTEMS ANALYSIS

by

W. A. McLaughlin
Graduate Assistant

Joint Highway Research Project

File No: 3-3-34

Project No: C-36-54HH

Purdue University

Lafayette, Indiana

May 18, 1965

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ABSTRACT

McLaughlin, Wallace, Alvin. Ph.D. Purdue University June 1965 Traffic Assignment by Systems Analysis. Major Professors: H. L. Michael, W. L. Grecco.

This research report is concerned with the assignment of traffic to a network of streets by systems techniques. Since the choice of route used by a traveller is not random, it follows that they use some general principles for route choice. A literature review of value theories and field studies governing route choice was undertaken. It was concluded from this review that various physical and psychological factors do govern the route choice made by individuals. However, a value function which would deterministically reflect the psychological factors subjectively used by the aggregate of travellers could not be determined. It was therefore postulated that cost of travel and time of travel would satisfactorily reflect the indeterminate value parameters used by an aggregate of travellers. Two types of value functions were used. One, involved a straight cost variable where cost included operating, accident, quality of flow and time costs. The other involved a variable that was a product of time and cost where the cost included all of the prior items except time. A relationship between speed and cost was developed such that a continuous value function in relation to flow could be employed.

A method was evolved such that paths or routes between any origin-destination pair could be determined. The basis of this path finding technique employs the empirical evidence available from previous diversion type studies. In essence, the method computes the "n" best paths in a network between any origin-destination pair subject to a diversion type restraint.

It is a hypothesis of this report that travellers will, under equilibrium conditions, distribute themselves such that between any origin and destination, the value function will be equal on the alternate paths developed by the path finding algorithm. The techniques of linear graph theory were used to assign traffic to the developed paths.

To evaluate the postulated value functions, path finding algorithm and linear graph assignment techniques, a synthetic network with synthetic loadings was assigned traffic by the various current techniques and compared to the assignments of the proposed algorithm. The proposed algorithm compared favourably with the other techniques.

The city of Brockville, Ontario was used to further evaluate the technique. Assigned volumes and ground counts were compared. The results showed that the value function which employed a straight cost variable would more precisely predict the traffic flow. The results also showed that the proposed algorithm predicted trips quite accurately.

INTRODUCTION

Traffic assignment is the process of allocating person or vehicular trips to an existing or proposed system of travel facilities. This process is invaluable from a transportation planning viewpoint in that it allows proposed facilities to be tested for traffic carrying ability before they are built. Further, the technique is used to evaluate and compare alternate travel systems.

Traffic assignment may be used as a completely independent operation whereby a trip table (traffic flow from all origins to all destinations) is known, or it may be linked to other phases of transportation planning such as trip distribution.

Traffic assignment techniques have advanced from the "judgment" stage through the "two-route" stage to the "network" stage. In the "two-route" analysis, assignment was made between one expressway path and one arterial street path for various origins and destinations. Diversion curves were formulated from empirical studies. These curves show the percentage of traffic split between an expressway path and an arterial street path based on such parameters as time ratio, distance ratio, or a combination of the two. Because of the obvious limitations of this technique a "network" approach has been adopted by most agencies responsible for transportation studies.

The network analysis considers assignment to the whole system. The method of allocation most commonly used is by means of

a "minimum path tree" whereby traffic is assigned to this minimum path on an "all-or-nothing" basis.* The "minimum path tree" is a series of connected roadways or links from an origin to all possible destinations which minimizes some travel function such as time, distance, cost, etc. All interzonal transfers are then assigned to these minimum paths. The most serious limitation of this technique is the "all-or-nothing" hypothesis. This hypothesis is not borne out by the empirical studies done to date.

To overcome this deficiency several "capacity-restraint" type solutions have been devised. These solutions fall into two distinct types. The first, applies a travel function as the network is loaded from successive minimum path assignments. The second applies tree building and all-or-nothing assignment to the whole network using a constant travel function. A capacity restraint is then applied to the whole network to take into account the original assigned volumes. New trees are then determined for the entire system based on the new constant travel function and reassignments made. These iterations are continued for a predetermined number of times or until a predetermined minimum difference in the travel function for each link is achieved.

The first type of capacity-restraint solution is computationally efficient but is not conceptually sound. The second type is more satisfying from a conceptual point of view but is computationally laborious.

The majority of assignment methods use travel time as an

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* See Appendix A for a list of definitions

index to reflect the users route choice. While this variable is important, it is probably not the only factor considered by the traveller.

Purpose and Scope

The purpose of this research was to develop an assignment technique which would overcome some of the conceptual and computational difficulties inherent in the present methods.

The study included an investigation of "value functions" which may serve as an indication of the principles which govern the route choice made by travellers. These functions were then used in the assignment technique. Linear graph theory was used as the basic method of assignment.

A synthetic network was chosen and assignments made by linear graph techniques were compared to assignments made by other techniques now in use.

A further evaluation of this technique was made by using a "real" system. The volumes assigned were checked against ground counts.

Only vehicular trips for a given trip distribution (constant trip table) were considered.

EXISTING ASSIGNMENT TECHNIQUES

Objective assignment techniques are a relatively new phenomena. One of the first attempts at an analytical solution was made by R. N. Brown (1)* in the late 1940's. Prior to this time assignment was carried out by "experienced" highway personnel. Since 1950, many methods have been developed and refined until all methods may be classified under three groups - judgment, two path analysis and network analysis.

In the judgment method, senior members of the highway department proportioned traffic between old and new facilities on the basis of their evaluation. Since this method is of limited use today, no further discussion of it will be presented.

The two path analysis considers assignment to one freeway route and one arterial route on a proportional basis. The travel or value function used for the selection of each route was on the basis of time, distance, cost or some function of one or more of these factors. In all but Brown's technique, the proportion of traffic allocated to a freeway was taken from a diversion curve. This method considers that the freeway will divert a certain percentage of the traffic from the arterial street. Induced traffic and growth traffic are considered for design purposes but do not enter into the percent diversion. The construction of these curves was based upon "field"

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* Numbers in parentheses refer to Bibliography.

studies.

The network analysis techniques consider the entire system (except local streets). This results in every link being considered for inclusion in the assignment process.

Two Path Methods

Indiana Method

Brown (1) published one of the earliest formulations of diversion assignments. It was explicitly based on distance but also implicitly considered time and speed.

The formula used was:

$$F = \frac{(F_1 + F_2)F_3}{100}$$

where: F = percent expressway use

F_1 = factor based on expressway distance

F_2 = factor based on access distance

F_3 = factor based on adverse distance

The "factors" were developed from field data, and the assumption of an average speed of 40 m.p.h. on the expressway and 20 m.p.h. on the arterial street. Further, it was assumed that the diversion on the basis of expressway and adverse distance varies parabolically while that of access distance varies linearly.

$$F_1 = 0 \qquad \text{for } a \leq 0.4 \text{ miles}$$

$$F_1 = 2.8a^2 + 30.24a - 11.65 \quad \text{for } 0.4 < a < 5.4 \text{ miles}$$

$$F_1 = 70 \qquad \text{for } a \geq 5.4 \text{ miles}$$

where: a = expressway distance, the length in miles of the expressway portion of the trip.

$$F_2 = 33.3 \frac{a}{a + b} - 3.3 \quad \text{for} \quad a > 0.4 \text{ miles}$$

$$\text{and} \quad 0 < b < 9a$$

where: b = access distance, the length in miles of the city street portion of the trip.

$$F_3 = 100 - 240 \left(\frac{v}{a}\right)^2 \quad \text{for} \quad a > 0.4 \text{ miles}$$

$$\text{and} \quad a + b - c = v$$

where: c = street distance, the total length of trip in miles by the most advantageous route.using only city streets.

Time Ratio Diversion Curves

In the 1950's, experimental studies were conducted to determine the relationship between proportional expressway usage and various parameters which might reflect those values used by the traveller for his choice of route. Among the factors considered were time ratio, distance ratio, cost ratio, length of trip, habit, purpose of trip, etc. However, certain parameters were ruled out.

"To be of practical value, for purposes of traffic assignment, a relationship must be established between tangible factors of influence and the usage of urban arterial highways. Travel time and travel distance qualify in this respect better than any others"(3).

These studies showed that a relationship did exist between the percent usage and travel time ratios or distance ratios; they also showed a relationship between percent usage when the absolute time and distance differentials were considered.

One of the earliest and most influential of these studies was reported by Trueblood (36). The value parameters considered were time ratio, distance ratio, the product of these ratios, absolute time

differentials and time ratio combined with length of trip. Except for the latter parameters, all relationships were expressed as a two dimensional array. Schuster (33) performed a multiple regression analysis of this data. His results are shown in Table 1. The parameter selected by Trueblood, the time ratio, also shows the best multiple correlation. The diversion curve developed from Trueblood's study is shown in Figure 1.

Cost Diversion Curves

At approximately the same period, investigations were conducted by May and Michael (25) to determine a diversion curve which would use more than one value parameter but still retain the simplicity of a two dimensional relationship. Value parameters of time and distance were lumped into a single cost parameter. The percent usage versus a cost ratio was then developed. This method appeared to give smaller dispersions from a central curve for the data analyzed than did the time or distance ratio methods.

Detroit Diversion Curves

The Detroit Metropolitan Transportation Study (12) was one of the first large scale studies of this type. As such, a thorough investigation of traffic assignment techniques was made. These investigations (4)(11) showed that a single value parameter such as time or distance ratio would not measure diversion within acceptable limits when applied to various geographic areas. Figures 2 and 3 show the results of these comparisons for the most extreme cases (4). To attempt to explain such differences in expressway usage between the various empirical studies other value parameters such as length of trip, trip times and speed

TABLE 1

Variability of Value Parameters

Parameter	R^2	Limits
Time Ratio	0.899	0.45 - 1.63
Distance Ratio	0.605	0.66 - 1.93
Time Differential	0.889	-6.7 - +8.6
Distance Differential	0.524	-2.7 - +1.5
All of the above	0.912	all of the above

Source: Reference 33

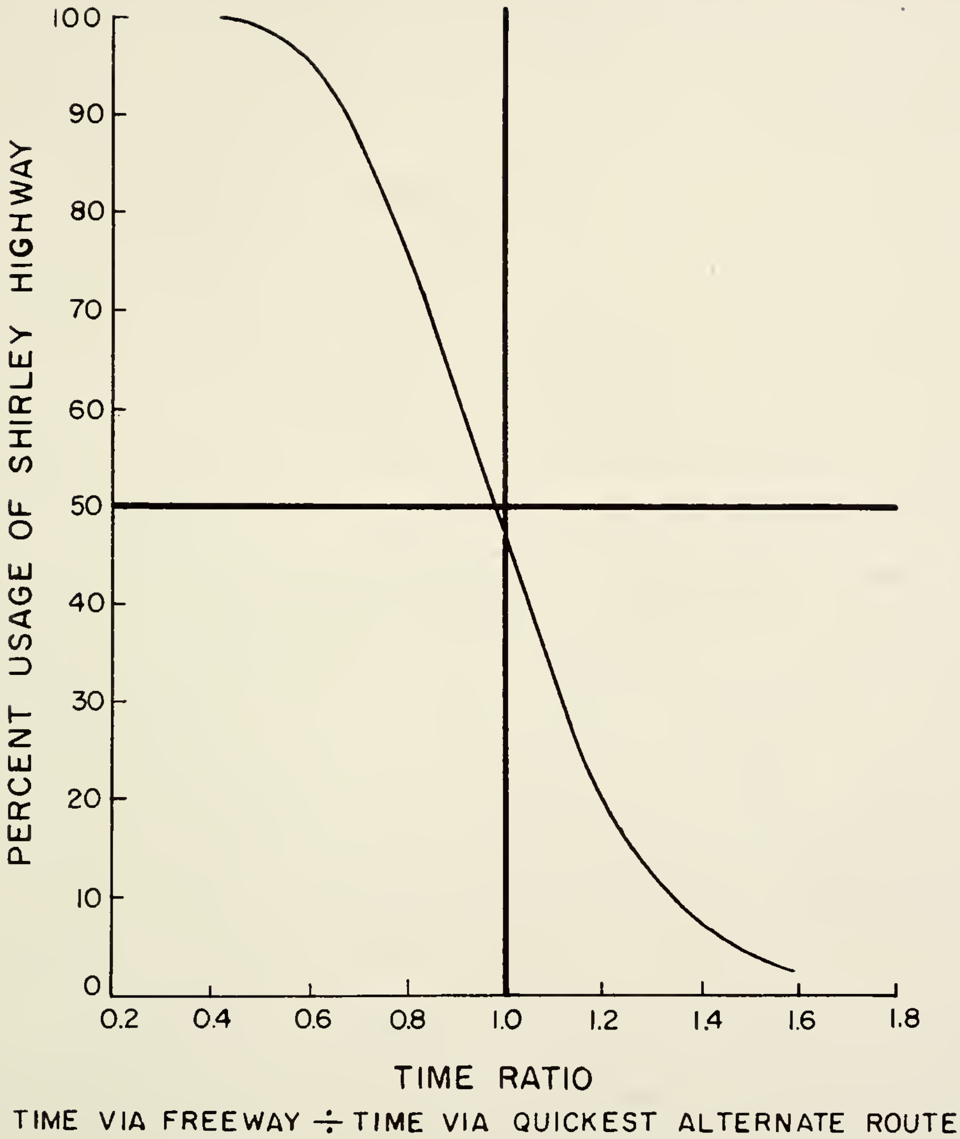


Fig.1 TRAFFIC DIVERSION CURVE USING TIME RATIO

SOURCE: REFERENCE 36

ratios were examined. It was logically deduced that these parameters would cause a variation in the diversion curves from location to location. Single value parameters of time differential and distance differential were examined and rejected. Since no one parameter seemed accurate enough to forecast traffic diversion, the Detroit group formulated a two parameter diversion surface. The first such formulation considered time and distance differentials. These parameters were chosen because of the available empirical studies made across the nation. In these studies, two methods were used - total trip and point of choice. Total trip surveys considered the total time via alternates from an origin to a destination. In the point of choice method measurements were only made for that portion of the trip which were not common. Time, distance and speed ratios will be different due to the method of study, but time and distance differentials are independent of the method of survey. Figure 4 shows the developed relationships. Although the variability in assignment was less by this two parameter formulation, it was discarded by the Detroit group because of the computational difficulties it entailed.

To obtain an assignment procedure which would be computationally efficient and at the same time consider the two value parameters of time and distance, the Detroit group evolved the distance ratio-speed ratio diversion curves. These curves were evolved from the Shirley study (36) since it was the only one made by the total trip method. These curves are thus not applicable to point of choice studies. Figure 5 shows the Detroit curves. The curves have a computational advantage over the time and distance differential curves if an assumption is made as to the ratio of speed of pure expressway travel to that of

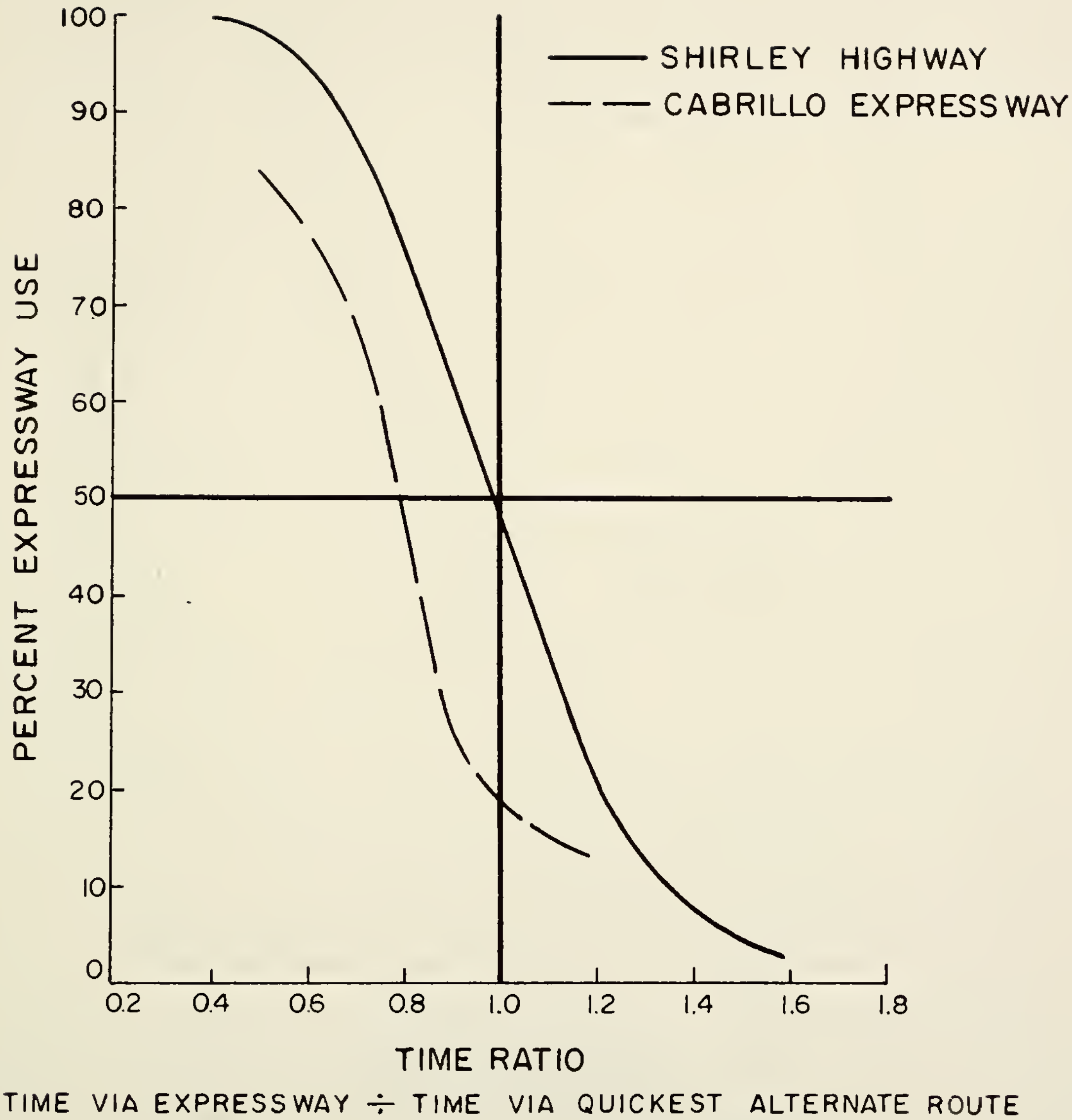


Fig. 2 COMPARISON OF TWO DIVERSION CURVES USING TIME RATIO

SOURCE: REFERENCE 4

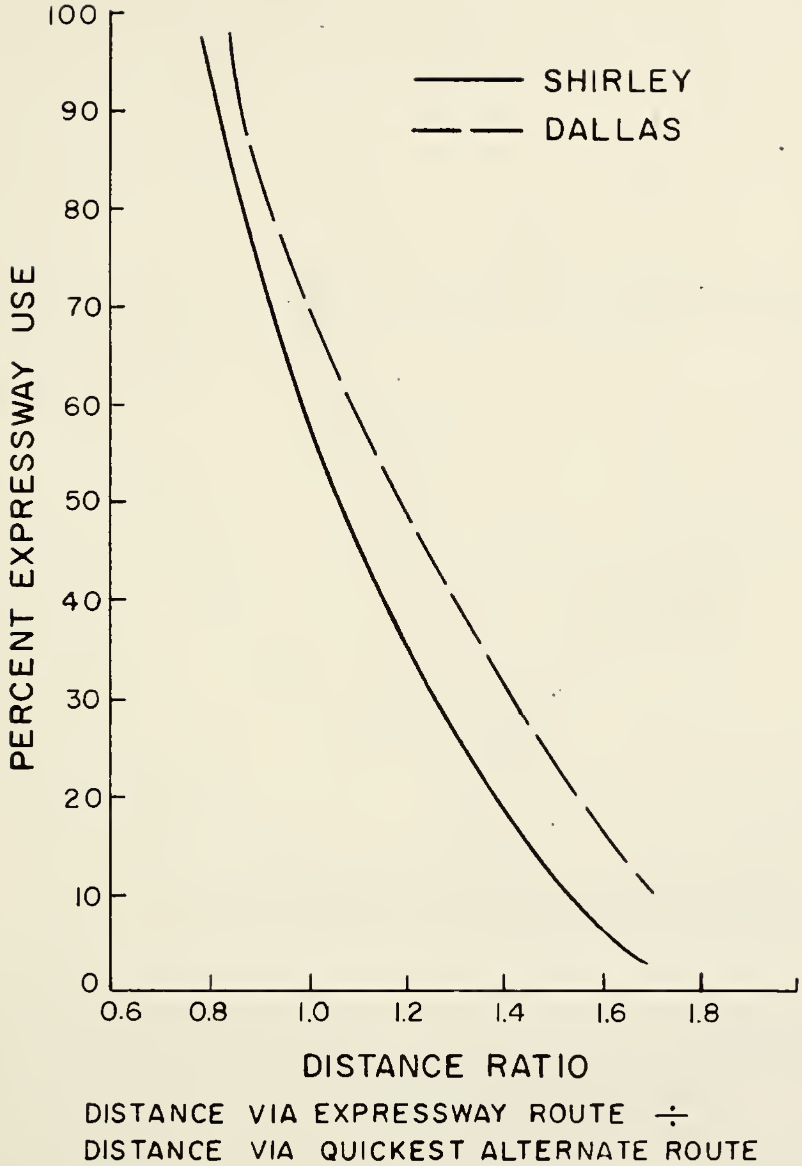


Fig. 3 COMPARISON OF TWO DIVERSION CURVES USING DISTANCE RATIO

SOURCE: REFERENCE 4

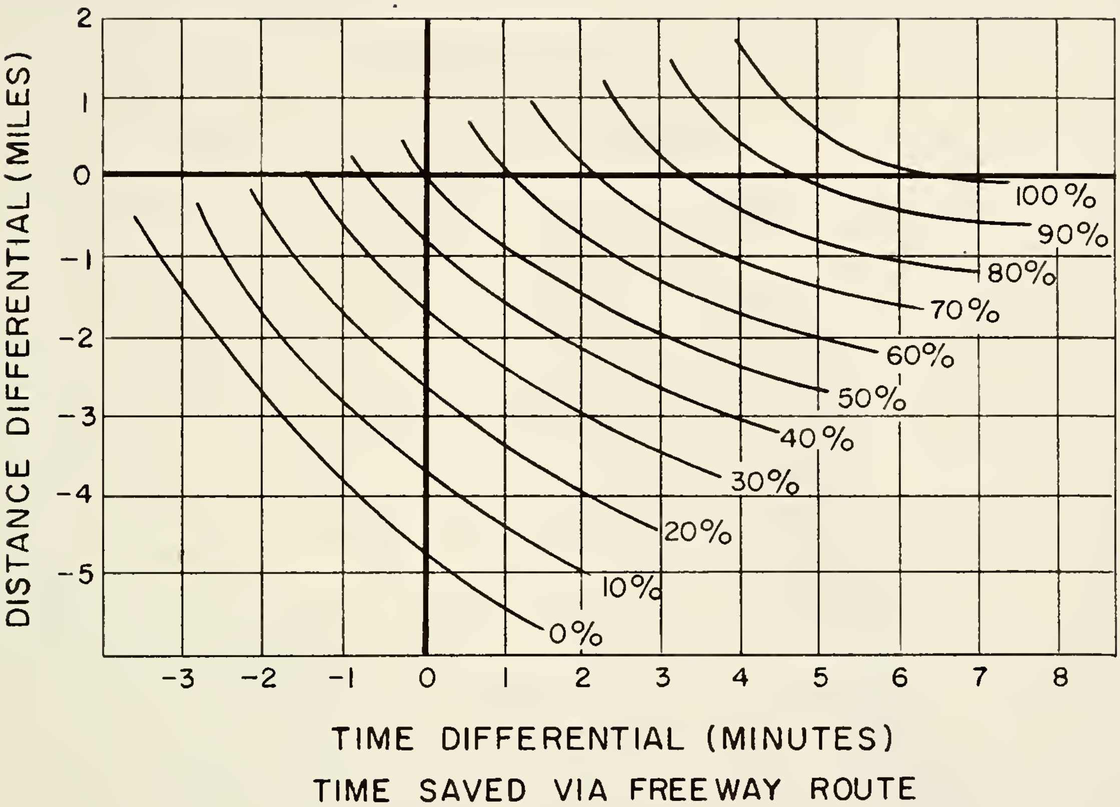


Fig. 4 INDIFFERENCE CURVES FOR PERCENTAGE EXPRESSWAY USE

SOURCE: REFERENCE 4

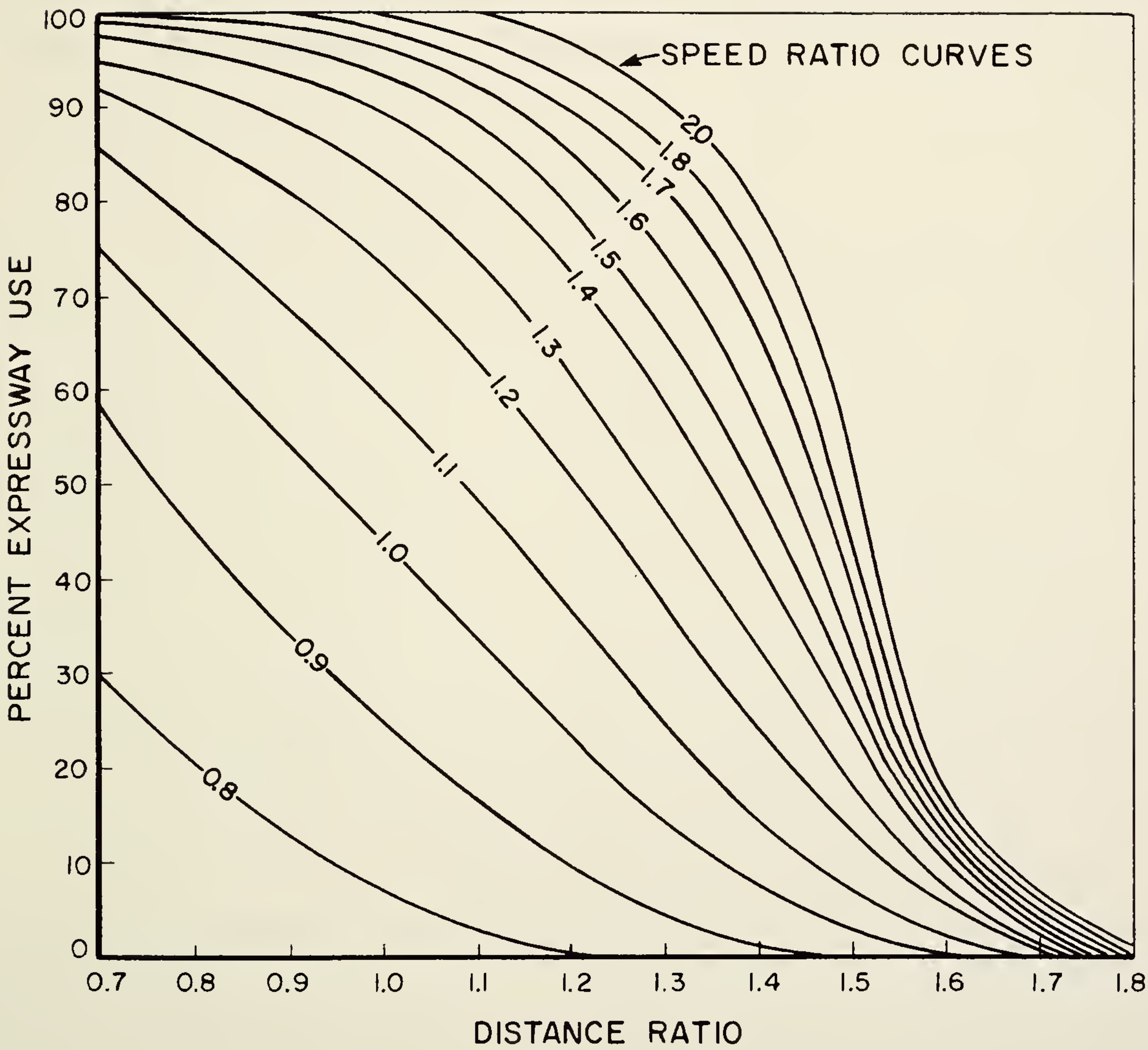


Fig.5 DETROIT DIVERSION CURVES

SOURCE : REFERENCE 12

city street travel. With this assumption only distances have to be measured on the alternate routes and proportional assignments computed from the curves. Hand assignments of zonal transfers are very lengthy calculations. As a result Detroit developed a machine procedure to handle these assignments (5). One assignment pass for the Detroit metropolitan region took three weeks.

California Diversion Curves

Studies in California (29) indicated that proportional diversion based on the single value parameter of time ratio did not yield adequate results for their planning purposes. From previous studies, they decided that the two value systems of time and distance differential would more accurately reflect diversion to the freeways. Studies were conducted on two freeways in California and indifference curves constructed. The results showed that iso-usage curves could not logically be constructed from the study points. Faced with this dilemma, the following assumptions were made and the diversion curves constructed (Figure 6).

1. Some motorists will drive any amount of distance to save time.
2. Some motorists will choose the shortest route regardless of the time consumed.
3. The usage curves have a hyperbolic shape, and they are symmetrical.
4. The more time saved, the greater the proportional usage.
5. The more distance saved, the greater the proportional usage.

The upper and lower boundaries of the curves were fixed on the basis of the above reasoning. The one hundred percent usage boundary appears in

the upper right hand quadrant. Any trips in this quadrant will save both time and distance. However, near the origin of the boundary (zero time and distance) motorists may not know of the saving. Hence, it was reasoned that the one hundred percent usage boundary should be plotted some distance from the zero axis. Because of the second postulate (a few motorists will choose the shortest route regardless of the time consumed) the 100% usage boundary could not cross the zero axis. The zero percent usage boundary was constructed in a similar manner. The proportional usages between the boundaries were assumed to be symmetrical and hyperbolic. The resulting equation was: (See Figure 6).

$$p = 50 + \frac{50(d + mt)}{\sqrt{(d - mt)^2 + 2b^2}}$$

where: p = percent usage freeway

d = distance saved via the freeway route in miles

t = time saved via the freeway route in minutes

m = slope of the 50% usage line

b = a coefficient determining how far the vertices of the 100% and zero percent boundaries are from the origin

Values of "m" and "b" were determined by trial and error from data covering two freeways in California. It was found that a reasonable solution existed when $m = 0.5$ and $b = 1.5$. The California diversion formula is thus:

$$p = 50 + \frac{50(d + 0.5t)}{\sqrt{(d - 0.5t)^2 + 4.5}}$$

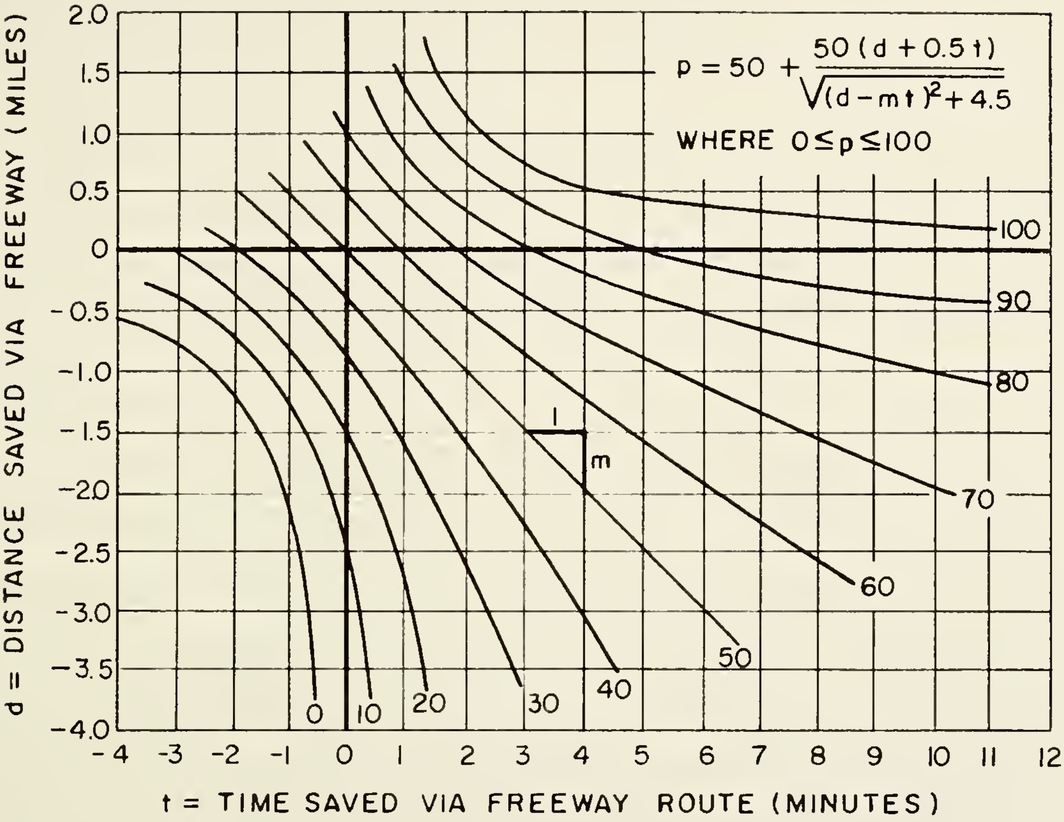


Fig. 6 CALIFORNIA INDIFFERENCE CURVES FOR PERCENT FREEWAY USEAGE

SOURCE: REFERENCE 29

Discussion of Two Path Assignment

Two path inter-zonal assignment using empirical diversion curves have obvious disadvantages. Only loadings on freeways are forecast. Further, only the "best" alternate route is considered in the assignment when in fact motorists will use the second, third, etc "best" routes. The diversion curve technique also makes use of a time or speed estimate based on existing traffic conditions. These values are used as the basis for assignment. Often when the assignment is completed, the assigned volumes bear no relationship to the initial assumption of time or speed. In addition, the assignment computations require a substantial time.

Although diversion curve methods were important and shed much light on the assignment process, they have other inherent weaknesses. These curves were based on studies with existing systems. Thus, the diversion rates are not only a function of the value parameters studied (time, distance, cost, etc.), but also of the capacities of the arterials and the freeways, and of the size of inter-zonal movements being considered. Hence, it is probable that the results of any one study would vary if the traffic pressures and/or capacities were different. However, for assignments to short sections of one freeway and one arterial street path, it may be quickest and best to use the existing diversion curves of California or Detroit.

Network Methods

Network methods of traffic assignment were evolved because of the inadequacies of the diversion curve or two path methods. These methods consider the total transportation network exclusive of local

streets. In most existing network methods an "all or nothing" assignment is made to a "minimum path tree" from one origin to every possible destination. New trees are constructed for every origin zone. This minimum path is usually expressed as a time function although cost, distance, effort or any value function could be minimized by this technique. Assignment by these techniques will generally result in traffic overload on some portion of the system and may require unreasonable road capacities to handle the assignment. This phase in the network assignment is usually termed the "unrestricted" or "demand" assignment. To attempt to simulate real life conditions many of the techniques employ a capacity restraint function which changes some of the minimum paths thus affecting assignments.

Chicago Method

The Chicago Area Transportation Study (6) (7) (8) pioneered the minimum path network assignment principles. Briefly, the assignment was made in the following manner.

- the loading or origin zones were selected in a specific ordering "thus preventing distortion and uneven loading due to the sequence of additing trips" (8). The method of ordering was not explained.

- from the first selected origin zone a minimum path tree, based on travel time at "free speed" to every destination zone, was constructed by Moores Algorithm (24).

- inter-zonal movements from this first selected zone were then assigned to this tree on an all or nothing basis without regard to capacity.

- the accumulated volumes on each of the loaded links were then compared to its capacity and new link times automatically computed from

the travel function derived for the Chicago study (time vs. volume to capacity ratio).

- for the second selected origin zone, using the revised link travel times, a new tree was calculated and an all or nothing assignment made.

- this process was repeated until all inter-zonal volumes had been assigned.

- demand or unrestrained assignment was achieved by constructing minimum path trees from every loading or origin zone using free speed time throughout the assignment process.

As in most restraint techniques one of the critical assumptions is that of the choice and functional relationship of the restraint or value function. The Chicago group selected time of travel as the value parameter for the assignment process. They then developed a functional relationship between speed and volume (hence time and volume) which could be used in their assignment process. Figure 7 shows the results of this study (6). The arterial curves are based on delays at signalized intersections and are standardized for one half mile link lengths and maximum discharge rates at the signal of 600 vehicles per hour.

The study showed that when the length of a link was greater than one half mile between signalized intersections and when the maximum discharge rate was greater than 600 vehicles per hour, the average speed of travel increased. However, to decrease the number of restraint formulas and to compensate for the neglect of acceleration and deceleration time losses the curves were standardized.

Two concepts of capacity were employed by the Chicago group. One was the "average maximum capacity" which was defined as "the average

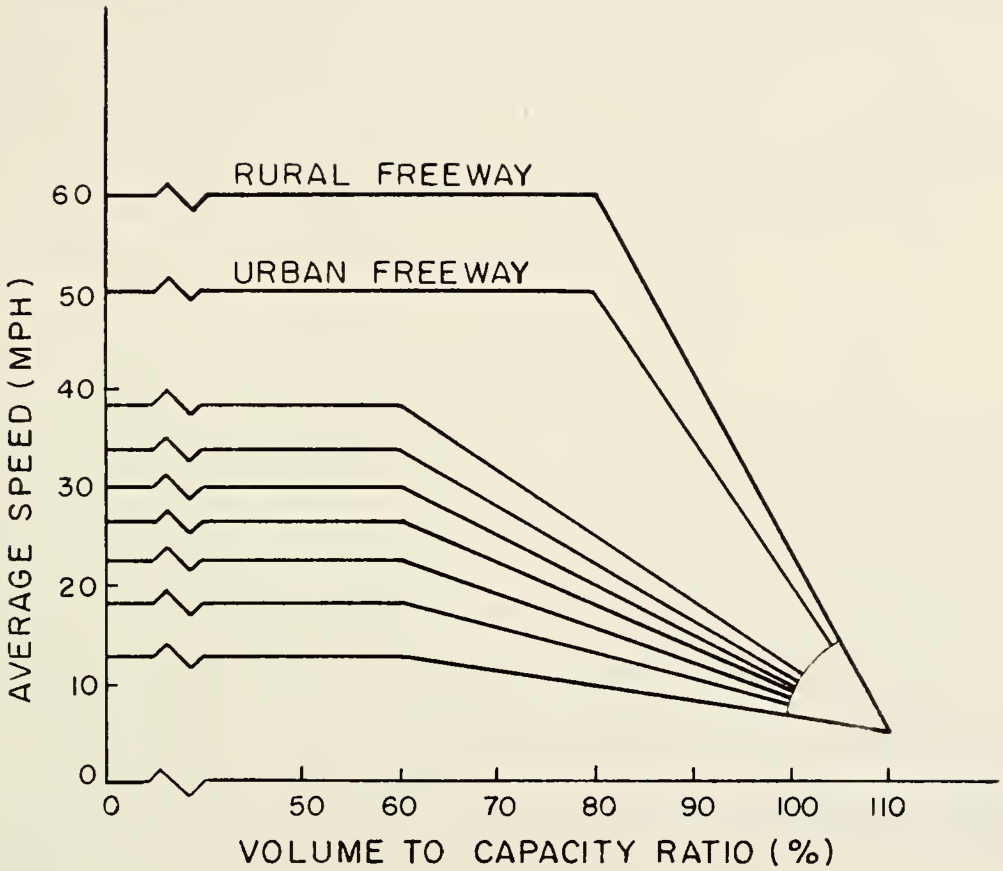


Fig.7 CHICAGO CAPACITY RESTRAINT FUNCTION

SOURCE: REFERENCE 8

maximum number of vehicles which can pass a point on a roadway in an hour"(8). The arterial street capacity figures were based on the discharge rate of vehicles through signalized intersections and hence are not affected by speed. The other concept was design capacity which was a reduction of average maximum capacity reflecting the quality of service concept. For rural and urban freeways, design capacity was taken as 85% of average maximum capacity. For arterial streets, the figure used was 70% of average maximum capacity. Table 2 shows the hourly average maximum capacities used in the Chicago study.

Pittsburgh Method

The method of assignment used in the Pittsburgh Area Transportation Study was similar to that used by Chicago. It differed in three respects. The loading or origin zones were selected randomly rather than orderly; the capacities used were the "practical capacities" as defined by the Highway Capacity Manual (19) rather than the average maximum capacity; the capacity restraint function (time vs volume to capacity ratio) changed. Table 3 shows the capacities used by Pittsburgh and Figure 8 shows the restraint function (31).

Wayne Arterial Assignment Method

This method utilizes a capacity function of an exponential form and assigns traffic to the various routes or paths between each origin and destination pair such that the travel times on these routes are all equal and the zero flow speed on any other route between the same origin - destination pair will have a larger time. It is an iterative procedure.

The procedure used is as follows: (33) (34) (35)

TABLE 2

Chicago Hourly Capacities for Streets

Average Maximum Capacities in Vehicles[a] per Hour

Arterial Streets By Type of Area	½ Pavement Width 10'	20'	30'
Down town (b)	480	1080	1800
Intermediate (b)	600	1320	2160
Outlying and Rural (b)	660	1440	2160
Expressways	2100 vehicles per hour per 12' lane		

(a) Expressed in Automobile Equivalents

(b) Assuming no parking; 50% green time; 10% right turns; 10% left turns

Source: Reference 8

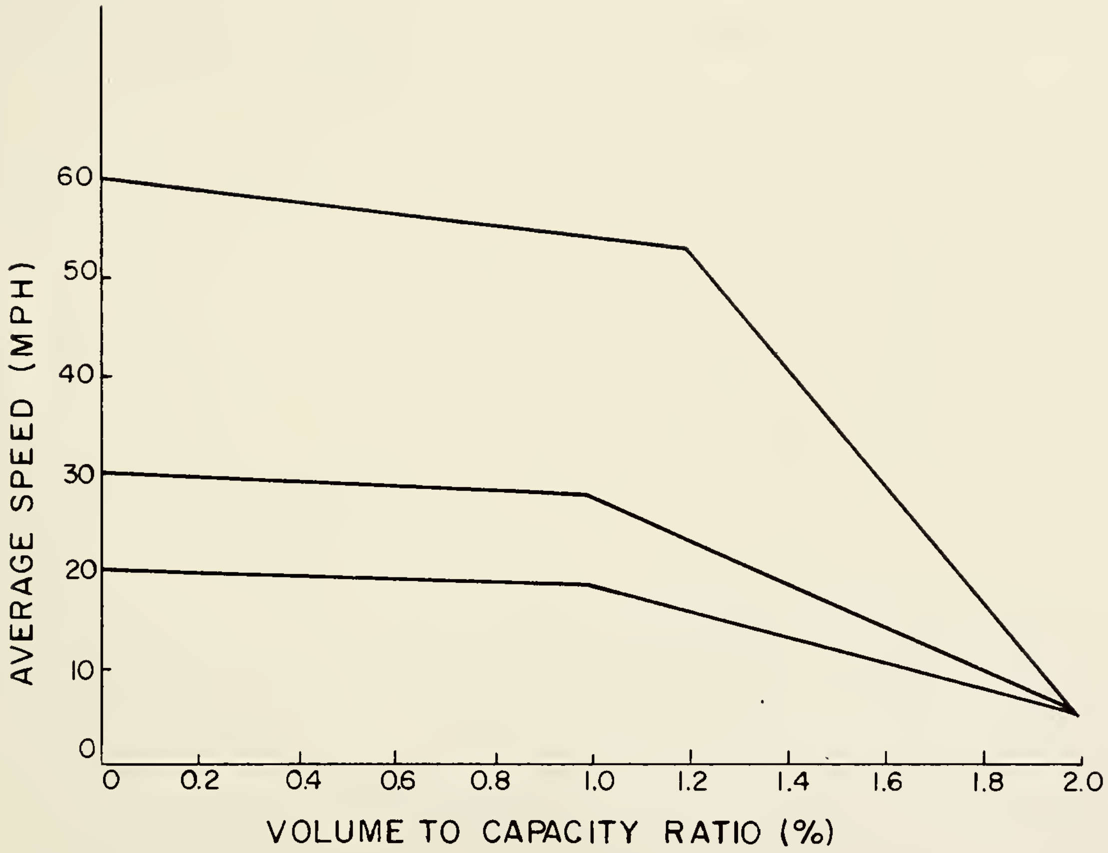


Fig. 8 PITTSBURGH CAPACITY RESTRAINT FUNCTION

SOURCE: REFERENCE 31

TABLE 3

Pittsburgh Hourly Capacities for Streets

Street Type	Approach Width (curb to center line)		
	10'	20'	30'
Arterial Downtown	280	560	840
Arterial Intermediate	400	800	1200
Arterial Outlying and Rural	500	1000	1500
Freeway all areas	1800 vehicles per hour per 12' lane		

Source: Reference 31

- minimum path trees are constructed for all origin zones based on travel times which are computed on the basis of average speeds under "typical" urban conditions (at practical capacity for all routes).

- inter-zonal volumes are assigned on all or nothing basis, without regard to an ordering of origin zones or link capacities. The accumulated link volumes reflect the "demand" or "desire" assignment.

- a capacity restraint formula is next employed to recalculate travel times on every link. This capacity restraint function is:

$$V_i = e^{(R_i - 1)} V_o$$

where: V_i = travel time on a link for a given iteration pass

R_i = ratio of averaged assigned volumes (from all preceding passes) to capacity

V_o = original ("typical") travel time on the link

- new minimum path trees are constructed for all origin zones based on these new calculated travel times. All links will have their travel times changed because of the form of the function. For those links not used, the travel times will decrease while for those links whose assigned volumes are greater than capacity, the travel times will increase.

- interzonal transfers are assigned to these new minimum paths on an all or nothing basis.

- the assigned volumes to each link are averaged for all iterations. This may be stated as:

$$\bar{X} = \sum_{i=1}^{n} \frac{X_i}{n}$$

where: $\bar{X}$ = average assigned link volume

X_i = trips assigned to the link during the ith iteration

n = number of iterations completed at any point in the program

- new link travel times are computed from the same capacity restraint formula with new values. That is:

$$V_3 = e^{(R_2 - 1)} V_o \quad \text{for the third iteration}$$

$$V_4 = e^{(R_3 - 1)} V_o \quad \text{for the fourth iteration}$$

where: V_3, V_4 = link travel times for third and fourth iteration respectively

V_o = original (typical) link travel time

R_2, R_3 = ratio of average assigned link volume ($\bar{X}$) to capacity after the second and third iteration respectively

- new minimum path trees are constructed and all or nothing assignments made for the interzonal transfers.

- the iterations are continued until balance occurs or until some pre-selected cutoff point is reached.

The capacity used in the restraint formula was defined as "the number of vehicles that can traverse the link under typical urban conditions including 10 percent signal failure at peak hour."(35) The arterial link capacity was estimated by averaging the capacities of the intersections at its ends.

The capacity function reflects normal conditions. That is, the effect on travel time is small when the flow is small and large when the flow is large. Again, as in other network methods, the "demand" flow can be any percentage of capacity since this excess demand over

capacity is reflected in high travel times due to queuing.

The averaging technique used in this method ensures that the travel times on each path between an O-D pair will reach equilibrium and hence converge to a constant value.

Traffic Research Corporation Method

This method of assignment combines trip distribution, modal split, and traffic assignment. The assignment phase, as in the previous network methods, uses Moores algorithm (24) to build minimum path trees. However, as the network is loaded up to nine different paths between any origin and destination zone may be developed. Traffic is split between the paths in proportion to the inverse of the travel times. The assignment procedure is continued until equilibrium is reached or until some "a priori" minimum difference is achieved. Travel time is also used in this method as the value parameter. The capacity restraint function relates travel time to vehicular flow.

The procedure used is as follows: (20) (21)

- trip generation is constant

- minimum path trees are found for all combinations of origin-destination zones on the basis of zero flow or free speed times. Up to four types of trees may be determined; one for private vehicles, one for transit vehicles, one for a mixture of private vehicles and transit and one for trucks. These routes are stored in "memory."

- based on the travel times between an O-D pair at free speed, time factors are calculated and the gravity model employed to generate a trip table. The travel times on these minimum paths are also used as one parameter in determining the modal split.

- a modal split is made between each O-D pair

- traffic is then assigned on an all or nothing basis to the respective minimum path trees (vehicular, truck, transit, combinations) and link volumes accumulated.

- the capacity restraint functions are then utilized to revise the link travel times

- new minimum time paths based on the revised link travel times are then constructed and stored in "memory."

- new modal split factors and a new trip table are determined for all zones.

- the revised interzonal interchanges by mode, are assigned to the minimum paths calculated up to this point in the procedure by the following formula:

For any one modal interchange (e.g. passenger cars)

$$J_{rij} = \frac{(T_{rij})^{-1} \; J_{ij}}{\sum_{r=1}^{n} (T_{rij})^{-1}}$$

where: J_{rij} = number of trips of a given mode going from origin i to destination j via r^{th} available route $(1 \leq r \leq n)$

n = number of routes available between i and j (two at this point in the procedure)

T_{rij} = travel time from i to j via the r^{th} available route

J_{ij} = total number of trips from i to j by a given mode

- these iterations are continued until equilibrium or some minimum difference in values is achieved. The assignment method, given

a constant trip table, could be used as an independent operation.

The capacity function in this model relating travel time to volume is based in part upon empirical evidence and in part from theoretical considerations. Seventeen types of functions are defined. The general equations describing the capacity functions are as follows:

For $0 \leq f(v) \leq f_c$: $t(v) = t_c + d_1 [f(v) - f_c]$

For $f_c \leq f(v) < f_m$: $t(v) = t_c + d_2 [f(v) - f_c]$

For $f_m < f(v)$: $t(v) = t_m + d_3 [f(v) - f_m]$

where:

$f(v)$ = vehicle demand flow in vehicles per hour per lane

$t(v)$ = average per unit vehicle travel time in minutes per mile

f_c = "critical flow" (near practical capacity)

t_c = average per unit vehicle travel time in minutes per mile at critical flow

f_m = maximum flow (possible capacity)

t_m = average unit vehicle travel time in minutes per mile at maximum flow conditions

d_1 = slope of the capacity function between 0 and f_c (the free flow region)

d_2 = slope of the capacity function between f_c and f_m (the turbulent region)

d_3 = slope of capacity function when the demand flow is greater than f_m (overload region)

Table 4 shows the "capacity table" used for the above formulation.

TABLE 4

Toronto Capacity Functions

Type	Speed Limit	Signals per mile	d_1	d_2	d_3	t_o	t_c	t_m	f_c	f_m
Cars	30	10	.0013	.0188	.0563	4.4	4.9	7.4	400	533
		5	.0011	.0167	.0500	3.4	3.9	6.4	450	600
		3	.0010	.0150	.0450	3.0	3.5	6.0	500	667
		1	.0008	.0125	.0375	2.3	2.8	5.3	600	800
Buses	30	10	.0013	.0188	.0563	4.4	4.9	7.4	400	533
		5	.0011	.0167	.0500	3.4	3.9	6.4	450	600
		3	.0010	.0150	.0450	3.0	3.5	6.0	500	667
		1	.0008	.0125	.0375	2.3	2.8	5.3	600	800
Street-cars	30	10	.0016	.0242	.0726	4.4	4.9	7.4	310	413
		5	.0014	.0208	.0625	3.4	3.9	6.4	360	480
		3	.0012	.0183	.0548	3.0	3.5	6.0	410	547
		1	.0010	.0147	.0442	2.3	2.8	5.3	510	680
Cars	40	2	.0007	.0100	.0300	1.9	2.4	4.9	750	1000
		1	.0006	.0083	.0250	1.7	2.2	4.7	900	1200
	50	1	.0005	.0068	.0205	1.5	2.0	4.5	1100	1467
		0	.0004	.0058	.0173	1.2	1.7	4.2	1300	1733
	60	0	.0004	.0054	.0161	1.0	1.5	4.0	1400	1867

Source: Reference 21

Linear Programming Methods (28) (30) (38)

These methods seek to establish traffic flows on a network in such a manner such that some travel function for all travellers in the system has a minimum value. Wardrop (36) in 1952 advanced this as the principle of overall minimization. This assignment technique implies regulation of traffic flow such that only those trips assigned to the various links will be able to use them.

The techniques in use vary with the assumptions made as to the functional relationship between the value parameters (i.e. time, cost, etc.) and traffic flow. To use the normal linear programming techniques, the relationship between the travel function and flow must be constant or a step function. If the relationship between the travel function and flow is continuous then minimization by Lagrangian methods is employed.

The general formulation of the Linear Programming Method is as follows (28):

$$\text{Minimize } \sum_{j=1}^{n} t_j \, Y_j^{\alpha} \text{ subject to:} \qquad (1)$$

$$\sum_{j=1}^{n} \epsilon_{rj} \, Y_j^{\alpha} = E_r^{\alpha} \quad r = 1, 2 \ldots\ldots m - 1 \qquad (2)$$

and $$\sum_{\alpha=1}^{P} Y_j^{\alpha} \leq c_j \qquad (3)$$

and $$Y_j^{\alpha} \geq 0 \qquad (4)$$

where:

t_j = travel time over link j (this is independent of flow)

Y_j = vehicular flow on link j

α = "copy." A copy associates all of the traffic flowing from or to a specified origin or destination. Thus, the equations are repeatedly solved for every origin or destination (not both) in the system.

n = number of links in the system

Equation (2) expresses Kirchoff's node condition for the α-th copy. That is, the net flow at any node is zero.

r = a particular node

m = number of nodes

ϵ_{rj} = the incidence number for the flow into the r^{th} node (+1 for input, -1 for output, 0 if the link is not connected to the node).

E_r^α = influx or efflux at the r^{th} node associated with the α-th copy.

P = the number of origins or destinations in the system

c_j = the capacity of link j

A variant of simplex procedures can be used to solve this system of equations.

Discussion of the Network Methods

Any traffic assignment method attempts to predict what traffic will use the various facilities in the future. Evaluation of the traffic carrying ability as well as economic analysis of the proposed network is therefore possible.

Two concepts have been employed in the assignment techniques to date. One is the allocation of traffic to specific routes on the basis of their desirability. This was commonly termed "assignment."

The definition of "demand" would more accurately reflect this allocation. That is, "demand" for a route or link is the number of vehicles per time unit allocated without any knowledge of the capacity of the links involved or the flow that will result on these links. This type of assignment is useful for planning purposes in that it shows the routes most travellers would like to use if real life limitations on capacity did not enforce re-routing. The other concept is commonly referred to as simulation or capacity restraint assignment. This type of analysis attempts to introduce more realism into the allocation procedure in that it is normally impractical to provide facilities to meet the demand allocation.

The Chicago and Pittsburgh methods share several points which are open to question. Both methods employ the all or nothing hypothesis. However, it is known that travellers will use several paths between any origin and destination pair. In addition, the selection of origin or loading nodes (whether random or systematically selected) may result in favouring those trips whose zones were first selected since free speed is used for the first selected loading. The minimum time paths as determined for the first few zonal assignments may not remain the minimum paths after all interzonal movements have been assigned. Further, the capacity restraint curves may not reflect actual travel conditions. For example, the Pittsburgh relationship for freeways (Figure 8) shows that at a volume of 2160 vehicles per hour per 12 foot lane, an average speed of 53 miles per hour can be maintained. For the same demand volume the Chicago curve (Figure 7) shows a speed of approximately 23 miles per hour. However, these methods are quick and computationally efficient for large networks.

The Wayne method utilizes a postulate by Wardrop (37) which states that in optimal assignment the time of travel between an origin-destination pair will be the same on all routes and less than the time of even a single vehicle on any other route between the same pair. This method obviates the difficulties inherent in the Chicago and Pittsburgh methods in the selection of loading zones. However, the capacity function used in this method is extremely sensitive. At very low link volumes the travel times change faster than they would under actual conditions.

e.g. $V_i = e^{(R_i - 1)} V_o$ (see page 26)

when: $R_i = 0$:

$$V_i = V_o e^{-1}$$

The "free speed" travel time is only 0.368 of the travel time based on the average speed under "typical" urban conditions. On a freeway with a "typical" speed of 50 m.p.h., the free speed would be approximately 136 m.p.h. At volumes near possible capacity, the change in travel time is not as rapid as that which occurs in real life. This sensitivity results in the development of minimum path trees and assignments to paths which would not normally carry any traffic for a particular inter-zonal interchange. By averaging the assignments from each iteration, these routes will ultimately balance out, but the process may require many iterations.

The Traffic Research Corporation method also avoids the difficulties of the method of selecting loading nodes by utilizing constant (zero flow) times on all links and assigning all inter-zonal movements to the respective minimum path trees. However, after utili-

zing the capacity restraint curves, the traffic is proportioned among all routes that have ever been assigned traffic on the basis of the reciprocal of travel times. There appears to be no theoretical proof that the iterations will converge. (11).

The Linear Programming methods assign traffic such that the total travel time on the whole system is a minimum. This implies enforcement. But, under normal circumstances, the driver acts as a free agent. Nevertheless, certain enforcement measures (ramp closures, one-way streets, reversible lanes) may be enacted to make this method more applicable to the real world. The results of the method may, however, be quite appealing to the planner in the sense that the assigned volumes may serve as a measure of optimality. One major disadvantage of this method is the necessary assumption that the relationship between travel time and volume on any link must be a constant or a step function.

All the methods use a value parameter of time. Time of travel is probably one of the most important factors affecting route choice. But, as indicated by the empirical studies of diversion curves, it is not the sole factor governing the behaviour of motorists.

SYSTEMS ENGINEERING CONCEPTS

General

A large part of the following discussion has been abstracted from Hall (18), Bross (1) and Churchman (9); for a more complete discussion of this topic the reader is referred to those texts.

Hall (18) defines systems engineering as the methodology underlying the solution to engineering design problems that arise from the needs and wants of society. Engineering design normally proceeds from needs analysis and feasibility studies through preliminary and detailed plans to plan effectation. In each of these steps there is a pattern of operations known as systems engineering.

No general theory of systems engineering exists, however, the structure of the process can be explained by six elements. These elements are briefly defined below:

1. Problem definition is the process of transforming an indeterminate situation into a pattern of factual data for formulating system objectives, synthesis and analysis. The environment within which a system must operate is not only the source of the need, but also the source of knowledge of every phase of the system engineering process.

2. Defining objectives is the terminal portion of problem definition and the formal definition of the desired physical system listing inputs, outputs and needs which the system aims to satisfy. The objectives are value statements and comprise the value system. The

logical function of this value system is to provide a means of judging the relative merits of alternative synthesized physical systems and to provide a criterion for specifying how the individual measures of value should be combined to arrive at a single value index for the system.

3. System Synthesis is the process of compiling a set of hypothetical systems which accomplish the objectives to a greater or less degree. The systems must be developed within the specified social, economic and technical constraints.

4. System analysis is the process of deducing all relative consequences of the alternative systems in light of the system objectives and constraints.

5. The selection of the optimum system is the decision to accept one of the alternate systems according to some criterion. This is a relatively simple problem when all value measurements are one dimensional (e.g. dollars) and made under certainty. It is very difficult when values are multi-dimensional (e.g. cost, safety) or made under uncertainty.

6. Performance Analysis is the procedure for assessing the serviceability of the implemented system in the "real world."

The definitions of the objectives of a system (design of the value system) is probably the most important area in engineering planning and design. It provides the means for optimizing systems and rules for choosing among alternates. All decisions involve a value system - usually an intuitive one such as good, bad, very poor, etc. Bross (1) states "there is a tendency for discussions of value to flounder and finally drown in a sea of platitudes." Unfortunately, there is no

general theory of value in existence. The following is a brief description of some special theories of value:

- The Causistic value theory holds that past decisions may be used to make present decisions. The causist therefore assumes that values are independent of time in the sense that if an identical problem can be found, the values and decisions made in the past can be applied to the existing problem. This theory of value is typical of decisions made by appeal to higher authority (e.g. building codes). In addition to the engineering profession, this system of reasoning is used by lawyers, theologians, urban planners and historians. The greatest weakness of this theory is the assumption that environment is static.

- The Economic Theory of value is concerned with the allocation of scarce resources among goods. Three concepts are used in this theory - market value, value-in-use, and imputed value. Money is the common denominator of the market value and imputed value concepts. It has the added advantage of being invariant under giving. These concepts are the ones most commonly used to reduce a multi-dimensional value system to one dimension. An example of this would be the value system of road user benefits (operating cost, time, safety, comfort, etc.) which are converted into a single measure of value. One of the technical flaws in this concept is the elasticity of the money unit. Value-in-use is an individual's subjective utility. It denotes the importance an individual places on an object or idea in relation to his own wants or needs. It is this situation of choice between alternates from which valuation arises; if an individual were not forced to choose between alternates no values could be placed on them. Market value is different from value-in-use in that market value reflects concensus

opinion whereas value-in-use is essentially personal. Furthermore, utility values are not transferable. Imputed value is an estimate of market value or an estimate of utility. Both market values and utilities are empirical concepts. The economic theory of value is quite definite compared to other theories of value.

- The psychological theory of value holds that value resides in any sort of interest or appreciation of an object or state of affairs. Thus, according to this theory, the measure of value is found in intensity of feeling. Psychological values exist in that they are embodied in the institutions of society. Thus, we find values classified as economic, moral, political, ethical, aesthetic and religious. It is difficult, but not impossible, to measure psychological values on some scale (i.e. opinion polls) but the use of this technique is limited to date. Direct questioning to establish a particular value scale involves several difficulties. The subjects may not be aware of any preference, or he may say what he imagines the interviewer would like to hear, or he may be misled by the question; or the subject won't cooperate. Direct observation of behaviour also has limitations. It has been shown that there is not a one to one correspondence between overt behaviour and attitude or feeling (13). Carefully prepared questionnaires by trained psychologists appear to be the best of the current methods of obtaining measures of attitudes or values. Another major difficulty in this theory of value is the measurement scale. Most psychological measurements are on the ordinal scale which for most decision processes are unsuitable. Further, intransitive ordering usually results when a conversion is made between the strength of individual preferences and the strength of group preferences.

The above three theories of value all suffer to some extent from a measurement point of view. For the general case of rational decision making value or utility functions must be measurable on the interval or ratio scale. Several attempts have been made by psychologists and economists to construct an interval or ratio scale of subjective values. However, these attempts are largely empirical and have met with only limited success.

Measurement may be defined as the act of assigning numbers to objects or events according to some set of rules. Three properties of numbers that are important to measurement are identity, rank order and additivity. Nine axioms are used to distinguish four levels of measurement: nominal, ordinal, interval and ratio scales. Table 5 lists the axioms and the classification of measurement scales:

Measurement problems, for rational decision making, are not resolved. Multi-dimensional values (i.e. cost, time, safety, aesthetics, etc.) must still be subjectively "traded-off" to arrive at a one dimensional index of merit.

In the systems engineering concept, synthesis and analysis requires some type of mathematical treatment. In general terms, these phases require the construction of a model which relates the topological properties of the system to the inputs and outputs of the system. Synthesis is the "idea-getting" stage; it involves the combination of parts to achieve a whole such that some objective is achieved. Most synthesis is done by interpolating or extrapolating existing techniques and results. These in turn are subject to analysis. Analysis is a separation of the system into components such that all consequences in terms of objectives are determined. Synthesis and analysis, in practice

TABLE 5

A Classification of Measurement Scales

Scale	Basic Empirical (a) Operations	Allowable Transformations	Examples
Nominal[b]	Determination of Equality	Any one to one Substitution	Catalogue Numbers
Ordinal[c]	Determination of greater or less	Any increasing monotonic function	Street numbers
Interval[d]	Determination of equality of intervals	$y = ax + b (a \neq o)$	Temperature (°F)
Ratio[e]	Determination of equality of ratios	$y = ax \ (a \neq o)$	Length, (°K)

a) the basic operations needed to create a given scale are those listed down to and including the operations listed opposite the scale

b) Identity Axioms: Either $A = B$ or $A \neq B$; if $A = B$ then $B = A$, if $A = B$ and $B = C$ then $A = C$

c) Rank Order Axioms: if $A > B$ then $B \not> A$; if $A > B$ and $B > C$ then $A > C$

d) Additivity does not exist unless an arbitrary zero is set

e) Additivity Axioms: if $A = P$ and $B > 0$ then $A + B > P$; $A + B = B + A$; if $A = P$ and $B = Q$, then $A + B = P + Q$; $(A + B) + C = A + (B + C)$

Source: References 9 and 18

cannot be separated and they are two faces of the same coin. Various techniques such as linear programming, critical path methods, queuing theory and graph theory are employed in this synthesis and analysis phase.

Value Synthesis in Transportation Planning

Traffic assignment is one facet to a decision process for the selection of a transportation network from a set of alternate networks. It is a sub-system of the field of transportation planning which is in turn a sub-system of urban or regional planning. Ultimately plans must reflect decisions made at various systems levels. Further, to be rational, the decisions must be consistent with the hierarchial objectives and the values placed on these objectives. Figure 9 shows a block diagram of the transportation planning system. The various planning activities (transportation, economic , social, etc.) in our society must reflect the wants or goals of that society. In choosing the objectives or the value synthesis in transportation planning several areas need detailed investigation. These areas include:

the environment - existing systems, acceptable technical standards, social and economic conditions, etc.

the needs - what does society want now and in the future? who will use the services?

measurement - how do we measure or place values on the various goals of the society group in relation to the total planning activity? how do we optimize the multi-dimensional system of value (social, economic, resource, etc.)?

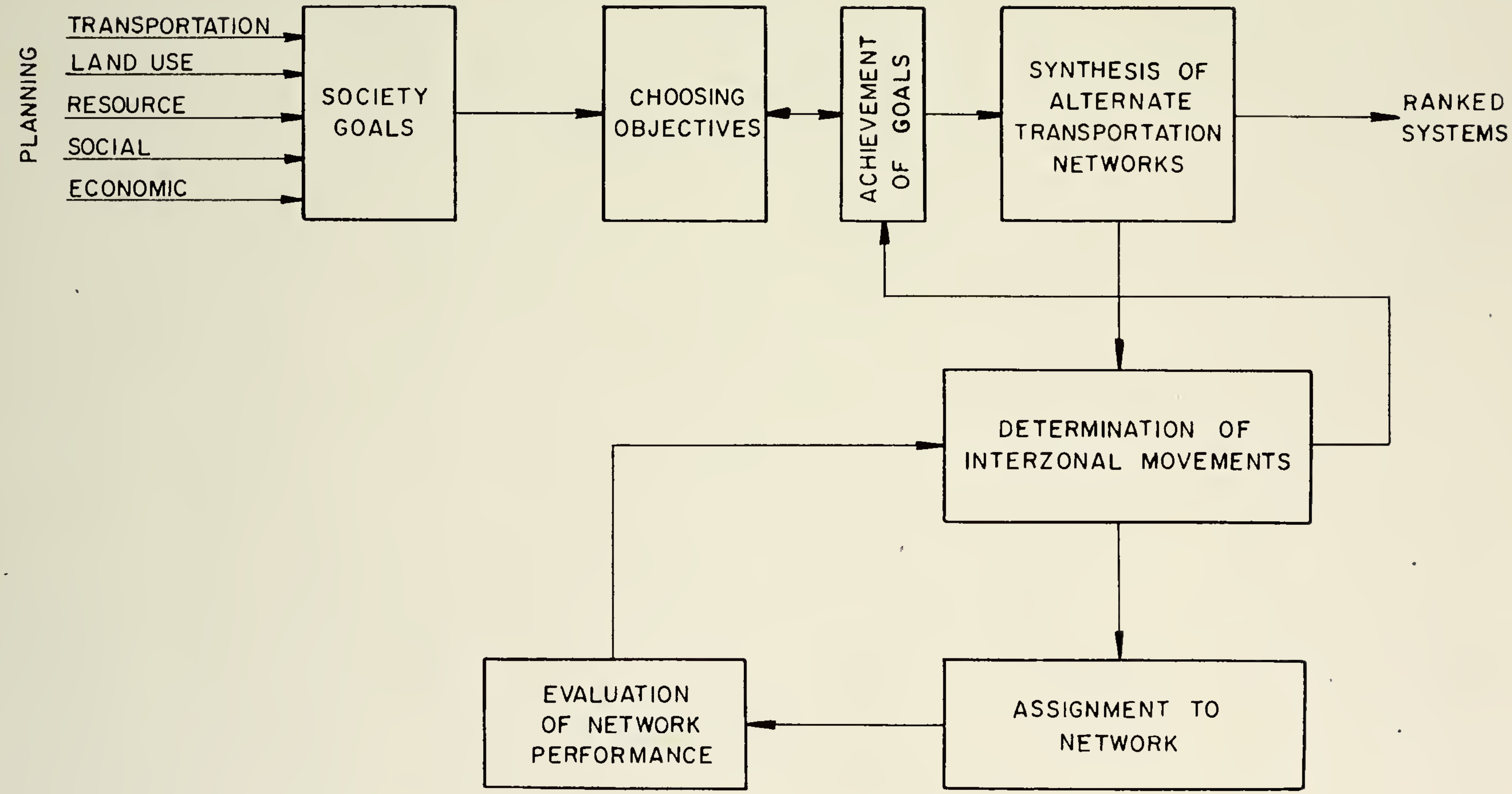


Fig. 9 TRANSPORTATION PLANNING SYSTEM

Environmental investigations have been resolved to handbook techniques. The needs and measurement areas, however, rest to a large extent on the causistic theory of value. In extreme situations, these areas are determined by an individual or a small group of individuals.

The chosen objective of transportation planning is usually given as:

> to develop an integrated system of transportation to provide an improved quality of service consistent with anticipated travel demands within the economic capabilities of the area and compatible with the requirements of the ultimate development of the area.

If this objective is accepted to be the society's objective, a problem of measurement to provide a rational decision criterion still exists.

The most commonly used operational objective in transportation planning is the least total cost solution coupled with a minimum attractive rate of return on investment. This objective minimizes the construction, maintenance and users costs of transportation networks. Other objectives of a social nature, since they cannot be measured on a ratio scale, are subjectively used in "trade-off" relationships.

Given the objectives of transportation planning, synthesis of various alternates must be made. Within the context of the given objectives, if the "best" alternate is not considered, the "best" solution will not be selected.

All alternate designs must be tested and evaluated. The primary tool for this phase is traffic assignment. Testing involves

the determination of the ability of the network to carry the traffic flux. Evaluation compares the performance, according to the measurable objectives, of a particular design to all other designs.

To distribute traffic over a network a hypothesis must be made about the objectives or values of the planner and/or the user. Under a free choice situation the traveller selects the route he uses. This implies that the user employs some value function which serves as the basis of his choice. The planner on the other hand may seek an allocation of trips which will minimize some value function which may or may not agree with the individual users sense of value.

In the two-path or diversion assignment method, allocation was made on the basis of what the motorist actually did under past measurement. In the minimum path methods the planner used his objective to minimize travel time on the system. This was more explicitly achieved in the linear programming methods.

It is the hypothesis of this thesis that travellers will, under equilibrium conditions, distribute themselves such that between any origin and destination, the value functions will be equal on the alternate paths. (Formulation of this value function will be discussed in the next chapter). Based on this hypothesis, the technique of graph theory to allocate traffic to a network is applicable.

Linear Graph Analysis*

The analysis made in this thesis is for two terminal components only. Hence, discussion will be limited to this type of system.

Graph techniques are based upon the premises that the complex

* For a complete discussion of this topic, refer to reference 22.

under investigation is a finite collection of discrete parts or components, united at a finite number of terminals and that the analysis is to be quantitative. Thus, mathematical models describing the components and their interconnections are required. The sequence for the solution of a physical system by linear graph analysis is shown in Figure 10.

In physical systems two fundamental variables are required to characterize the various phenomena. (Thermal, electric, hydraulic, etc.). These variables have been termed the "through" or "y" and the "across" or "X" variables. This terminology arose from instrumentation when measurements were made in "series" (through variable) and in parallel (across variable). The characteristics of a component are completely described if a measurable functional relationship between X and y can be obtained. This relationship is called the terminal characteristic of the component. An oriented line segment corresponding to the measurements on the component is known as the terminal graph of the component. The quantitative functional relationship between X and y is the mathematical model of the component.

The performance of a system depends not only on the individual components but also in the way they are connected. An interconnection model is also necessary before a solution can be obtained. If the terminal graphs of a set of components are interconnected in a one to one correspondence with a union of physical components, the result is a collection of line segments known as a systems graph or oriented linear graph. The interconnection model is described by two basic postulates. One, the <u>vertex postulate</u> states that at any vertex (V):

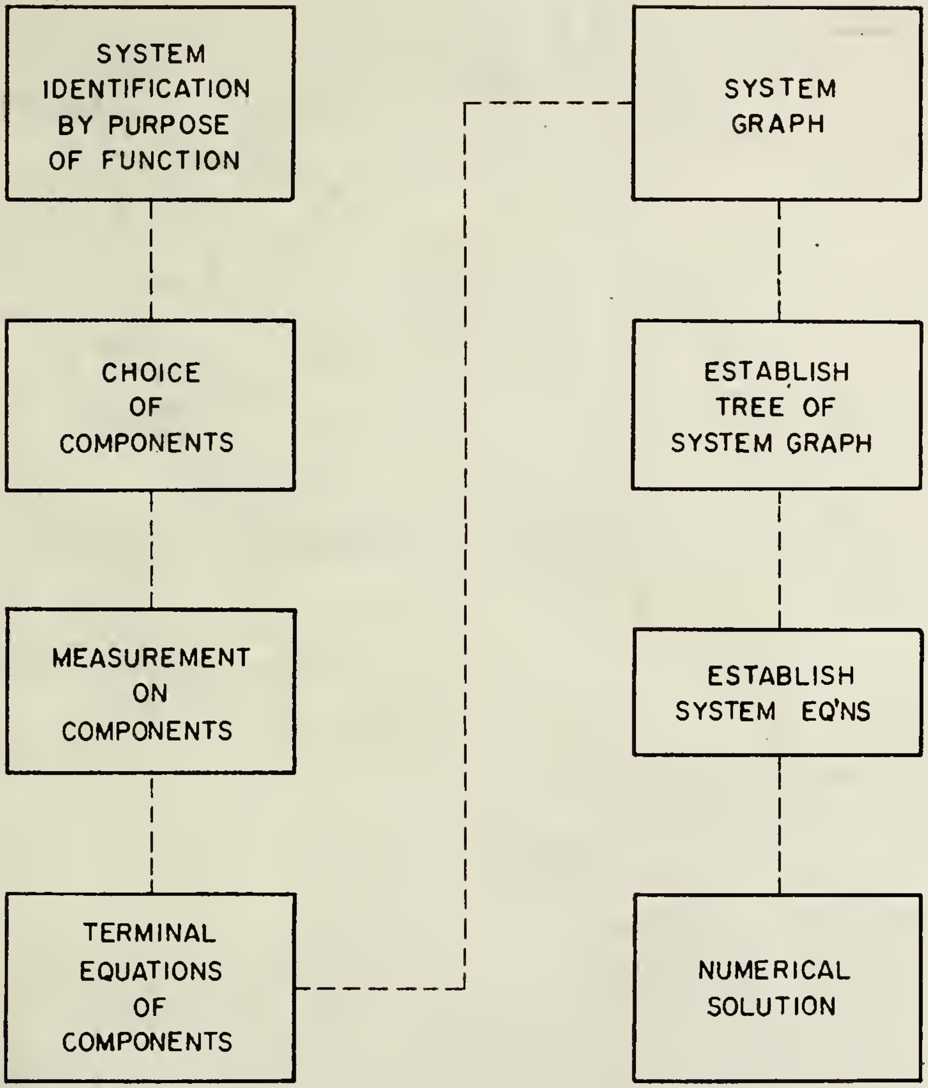


Fig. 10 SEQUENCE FOR THE SOLUTION OF A PHYSICAL SYSTEM BY LINEAR GRAPH ANALYSIS

SOURCE: REFERENCE 14

$$\sum_{i=1}^{e} a_i y_i = 0$$

where: e = number of oriented terminal graphs or elements

y_i = "Through" variable of the i^{th} element

a_i = 0 if the i^{th} element is not incident at the V^{th} vertex

a_i = 1 if the i^{th} element is oriented away from the V^{th} vertex

a_i = -1 if the i^{th} element is oriented toward the V^{th} vertex

The other, known as the circuit postulate states that for any circuit in the system graph:

$$\sum_{i=1}^{e} b_i X_i = 0$$

where: e = the number of elements in the graph

X_i = across variable of the i^{th} element

b_i = 0 if the i^{th} element is not in the j^{th} circuit

b_i = 1 if the orientation of the i^{th} element is the same as the orientation chosen for the j^{th} circuit

b_i = -1 if the orientation of the i^{th} element is opposite to that of the j^{th} circuit

These postulates are the familiar Kirchoff current and voltage laws for electrical networks or Newton's first law and the compatibility law in mechanics.

Any system can be solved by use of the terminal equations, the vertex equations and the circuit equations. However, not all of these equations are independent. To select the minimum number of in-

dependent equations, the concepts of fundamental circuit and cutset equations are used.

A fundamental circuit of a graph for any selected tree (the formulation tree) is the set of circuits formed by each chord and its unique tree path. The number of independent circuit equations is given by the product of the circuit matrix and the column matrix of the across variables.

$$\begin{bmatrix} B_{11} & u \end{bmatrix} \begin{bmatrix} X_b \\ X_c \end{bmatrix} = 0$$

where: B_{11} is a coefficient matrix corresponding to the branches

u is a unit matrix corresponding to the chords

X_b is the column matrix of the branches

X_c is the column matrix of the chords

The fundamental circuit matrix $B = [B_{11} \quad u]$ is defined by

$B = b_{ij}$ where:

$b_{ij} = 1$ if element j is in the circuit i and the orientation of the circuit and the element coincide

$b_{ij} = -1$ if the element j is in circuit i and the orientations do not coincide

$b_{ij} = 0$ if the element j is not in circuit i

The order of this matrix is $(e - v + 1)$, e

where: e = the number of elements

v = the number of vertices

The fundamental set of cut sets with respect to a tree is the cut sets formed by each branch of the tree and all cords of the tree for which the fundamental circuit (with respect to the tree) contains

this branch. The number of independent cut set equations is given by the matrix product of the cut set matrix and the column matrix of the through variables.

$$[u \quad a_{12}] \begin{bmatrix} Y_b \\ Y_c \end{bmatrix} = 0$$

where: u is a unit matrix corresponding to the branches of a tree

a_{12} is a coefficient matrix corresponding to the chords

Y_b is a column matrix of the branch through variables

Y_c is a column matrix of the chord through variables

The fundamental cut set matrix $a = [u; a_{12}]$ is defined by $a = a_{ij}$

where: $a_{ij} = 1$ if element j is in cut set i and the element orientation and the orientation of the defining branch coincide

$a_{ij} = -1$ if element j is in cut set i and the element orientation and the orientation of the defining branch do not coincide

$a_{ij} = 0$ if element j is not in cut set i

The order of the cut set matrix is $(V - 1),e$

If a tree is selected from a graph and the fundamental circuit and cut set matrices are formed with the columns of $[B]$ and $[a]$ arranged in the same order, it may be shown that (22):

$$aB^T \equiv 0 \quad \text{or} \quad Ba^T \equiv 0 \text{ whence } a_{12} = -B_{11}^T \text{ and } B_{11} = -a_{12}^T$$

where: a = fundamental cut set matrix

B^T = transpose of the fundamental circuit matrix

a_{12} = coefficient matrix corresponding to the chord through variables

B_{12} = coefficient matrix corresponding to the branch across variables

Thus, the fundamental cutset matrix corresponding to a tree can be written from the fundamental circuit matrix of the same tree, and conversely.

Chord Formulation

The chord formulation for the analysis of the system graph is the technique used in this thesis, and hence only this formulation will be presented.

The analysis of the system is based on the establishment of the terminal equations, the fundamental cut set equations and the fundamental circuit equations. The formulation requires that the given across variables (across drivers) be placed in the branches (X_{b-1}) and that the given through variables (through drivers) be placed in the chords (Y_{c-2}). It is also required that the terminal equations be given explicitly in the across variables.

From any selected tree, the fundamental circuit equations can be represented, symbolically as:

$$\begin{bmatrix} B_{11} & B_{12} & u & 0 \\ B_{21} & B_{22} & 0 & u \end{bmatrix} \begin{bmatrix} X_{b-1} \\ X_{b-2} \\ X_{c-1} \\ X_{c-2} \end{bmatrix} = 0 \qquad (1)$$

The terminal equations are expressed explicitly in terms of the across variable as:

$$\begin{bmatrix} X_{b-2} \\ \\ X_{c-1} \end{bmatrix} = \begin{bmatrix} R_{11} & 0 \\ \\ 0 & R_{22} \end{bmatrix} \begin{bmatrix} Y_{b-2} \\ \\ Y_{c-1} \end{bmatrix} \qquad (2)$$

where: R_{11} and R_{22} represent a coefficient matrix.

Expanding equation (1) such that X_{b-2} and X_{c-1} are in a separate column shows:

$$\begin{bmatrix} B_{11} & 0 \\ \\ B_{21} & u \end{bmatrix} \begin{bmatrix} X_{b-1} \\ \\ X_{c-2} \end{bmatrix} + \begin{bmatrix} B_{12} & U \\ \\ B_{22} & 0 \end{bmatrix} \begin{bmatrix} X_{b-2} \\ \\ X_{c-1} \end{bmatrix} = 0 \qquad (3)$$

The terminal equations (2) are substituted into (3)

$$\begin{bmatrix} B_{11} & 0 \\ \\ B_{21} & u \end{bmatrix} \begin{bmatrix} X_{b-1} \\ \\ X_{c-2} \end{bmatrix} + \begin{bmatrix} B_{12} & u \\ \\ B_{22} & 0 \end{bmatrix} \begin{bmatrix} R_{11} & 0 \\ \\ 0 & R_{22} \end{bmatrix} \begin{bmatrix} Y_{b-2} \\ \\ Y_{c-1} \end{bmatrix} = 0 \qquad (4)$$

One of the advantages of this type of analysis is the possibility of replacing certain unknown variables in an equation with known variables. In this formulation Y_{b-2} is expressed in terms of the chord through variables Y_{c-1} and Y_{c-2}. This relationship is obtained from the fundamental cut set equations.

$$\begin{bmatrix} u & 0 & -B_{11}^T - B_{21}^T \\ \\ 0 & u & -B_{12}^T - B_{22}^T \end{bmatrix} \begin{bmatrix} Y_{b-1} \\ Y_{b-2} \\ Y_{c-1} \\ Y_{c-2} \end{bmatrix} = 0 \qquad (5)$$

Expanding (5), we have:

$$\begin{bmatrix} u \\ \\ 0 \end{bmatrix} \begin{bmatrix} Y_{b-1} \end{bmatrix} + \begin{bmatrix} 0 \\ \\ u \end{bmatrix} \begin{bmatrix} Y_{b-2} \end{bmatrix} + \begin{bmatrix} -B^T_{11} - B^T_{21} \\ \\ -B^T_{12} - B^T_{22} \end{bmatrix} \begin{bmatrix} Y_{c-1} \\ \\ Y_{c-2} \end{bmatrix} = 0 \qquad (6)$$

Taking the bottom set of equations of (6) and including the identity $Y_{c-1} = Y_{c-1}$ yields:

$$\begin{bmatrix} Y_{b-2} \\ \\ Y_{c-1} \end{bmatrix} = \begin{bmatrix} B^T_{12} & B^T_{22} \\ \\ u & 0 \end{bmatrix} \begin{bmatrix} Y_{c-1} \\ \\ Y_{c-2} \end{bmatrix} \qquad (7)$$

Substituting (7) into (4) the form of the equations are:

$$\begin{bmatrix} B_{11} & 0 \\ \\ B_{21} & U \end{bmatrix} \begin{bmatrix} X_{b-1} \\ \\ X_{c-2} \end{bmatrix} + \begin{bmatrix} B_{12} & U \\ \\ B_{22} & 0 \end{bmatrix} \begin{bmatrix} R_{11} & 0 \\ \\ 0 & R_{22} \end{bmatrix} \begin{bmatrix} B^T_{12} & B^T_{22} \\ \\ U & 0 \end{bmatrix} \begin{bmatrix} Y_{c-1} \\ \\ Y_{c-2} \end{bmatrix} = 0 \qquad (8)$$

or:

$$\begin{bmatrix} B_{11} & 0 \\ \\ B_{21} & U \end{bmatrix} \begin{bmatrix} X_{b-1} \\ \\ X_{c-2} \end{bmatrix} + \begin{bmatrix} Z_{11} & Z_{12} \\ \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} Y_{c-1} \\ \\ Y_{c-2} \end{bmatrix} = 0 \qquad (9)$$

where: Z_{11} etc. is a coefficient matrix of the matrix triple product.

Using the first line of (9) a solution for the unknown through variables is obtained:

$$[B_{11} \cdot X_{b-1}] + [Z_{11} \cdot Y_{c-1}] + [Z_{12} \cdot Y_{c-2}] = 0$$

$$Y_{c-1} = Z^{-1}_{11} [-Z_{12} \cdot Y_{c-2} - B_{11} \cdot X_{b-1}]$$

where: Y_{c-1} are the unknown through variables

Y_{c-2} are the specified through drivers

X_{b-1} are the specified across drivers

The remaining unknowns may be solved from the cut set, circuit and terminal matrices.

SYSTEMS CONCEPTS APPLIED TO TRAFFIC ASSIGNMENT

General

This thesis is concerned with vehicular assignment to a network of streets. Although it is desirable to consider traffic assignment in conjunction with trip distribution and modal split, to keep the study within bounds, it will be assumed that for any network the trip distribution (i.e. trip table) is constant. Since the method will be checked against an existing network, this assumption, for the present day, is valid.

The formulation will follow the steps as shown in Figure 10.

1. System Identification by Purpose or Function
2. Choice of Components
3. Measurement on Components
4. Terminal Equations of Components
5. Systems Graph
6. System Equations

System Identification

The object of this research is to determine the demand assignment and/or the simulation assignment for each link in a street network given a trip table. The structural makeup of the system is, therefore, composed of trip inputs from each origin zone, trip outputs at each destination zone corresponding to the inputs, and a street system joining each origin-destination pair.

Choice of Components

The choice of components for a system is dependent on the purpose and structure of the system under study. Further, any component selected must be conceptual, definitive and quantitatively descriptive on a ratio or interval scale.

Since a trip table is given, one component is the number of trips from the centroid of any origin zone to the centroid of any destination zone. This component meets the three requirements stated above.

The other basic component is the street and its intersections. As in other assignment methods, local streets are not considered in the network. This exclusion is made since it is assumed that local streets carry only intra-zonal movements which are not considered; there is no congestion problem on this type of street; their inclusion would enlarge the network beyond manageable proportions.

Measurements on Components

If the techniques of linear graph theory are to be used in the system solution, the following requirements must be met:

1. The components must be describable, mathematically, by two fundamental variables
2. When the components are arranged in a system graph, one of the measured variables, X, must sum to zero around a circuit, the other variable, y, must sum to zero at the vertices of the graph.
3. The X and y measurement must be related by a linear or non-linear function.

The most logical y measurement for traffic assignment would be traffic flow. In other physical phenomena (electric, hydraulic, etc.), the y measurement represents flow. For traffic assignment this variable would satisfy the vertex postulate.

In other physical phenomena, the X measurement in some type of pressure differential that caused the flow. For this system (i.e. traffic assignment), it is hypothesized that travellers assign some value when making a choice of a route and that, under equilibrium conditions, the values will be equal for alternate paths. The reasoning for this hypothesis is as follows:

1. If it is believed that traffic can be assigned or simulated to specific routes or links with reasonable reliability, then it follows that some general principles govern the choice of route used by the traveller. Or, stated another way, there is some basis of variation for the flow of trips to alternate paths.
2. The individual user will act as a free agent and seek to optimize his value
3. Under stable conditions, the aggregate values, X, will be equal for the alternate paths between any pair of origin-destination zones.

This "pressure" term can best be described as a function of other factors which explain the variation in flow.

Terminal Equations of Components

The terminal equations of the components have been assumed to be of the form:

$$X = R(y) \cdot y$$

where:

1. The y value is the flow of vehicular trips. This value is specified for the trip table component.
2. The R value is the resistance to flow. This value is postulated as the product of the time per vehicle and the cost per vehicle to traverse a link at any particular flow. Or the total cost (including time) to traverse a link at any particular flow.
3. The X value shall be a postulated measure of imputed value (cost per vehicle) that travellers use in selecting a route.

Subjective Values Used by Travellers

Several studies (4) (29) have been undertaken to determine the subjective values travellers use when selecting alternate routes. These studies are, like many psychological investigations, qualitative in nature. In addition, these studies only covered subjective values for a choice between a freeway route and the "best" alternate arterial route.

Table 6 shows the results of a study reported by Campbell, based on a free or open end type of query, (4). Seventy one percent of the 107 interviews gave emphasis to time or distance. Arterial users gave predominantly distance oriented reasons for route choice while time oriented reasons predominated the expressway route choice. The travellers

TABLE 6

Detroit Study of Subjective Travel Values

Advantage of Chosen Route	Total	Expressway User	City Street User
Distance Oriented Advantages	42	8	34
Time Oriented Advantages	33	26	7
Traffic and Traffic Movement (Less Traffic, fewer controls)	17	7	10
Road Characteristics	4	0	4
Miscellaneous (habit, safer, fewer turns) no answer	11	3	8
TOTAL	107	44	63

Source: Reference 4

perception of time and/or distance was also studied. Combining the perceptions of time and distance 41 out of 107 drivers were correct in their perceptions of time and distance. In addition, 58 (of 107) were correct in one dimension. The remainder were indeterminate.

A study conducted in California and reported by Moskowitz is shown in Table 7 (20). This investigation again shows that time and distance factors predominate when an open-ended question was asked, particularly when time and distance differentials were relatively large. When time and distance differentials favoured the arterial route other values seemed to predominate. Again, arterial users gave predominantly distance values whilst freeway users favoured time values.

These studies indicate, in a qualitative manner, the complexities which are involved in the individual value judgments of the motorist. Further, as discussed under value theories, it is almost impossible to construct a ratio or interval scale which would measure the aggregate values of the users. In the two path methods, it was concluded by the investigators at that time that although other values did influence the traveller, objective factors such as time ratio, speed ratio, distance ratio, time and distance differentials could be used to reflect the value judgments of the motorist. Most network methods used time alone as the objective value parameter.

Because of these measurement complexities and the need for objective scales it was concluded that a value function based on psychological factors could not be constructed at this time.

It is a postulate of this thesis that objective value parameters of time and cost would satisfactorily reflect the indeterminate subjective value parameters used by an aggregate of travellers. It is

TABLE 7

California Study of Subjective Travel Values

Number of people giving reasons for using either freeway or alternate route

Distance Saved via Freeway (miles)	$d > \frac{1}{2}$		$0 \leq d < \frac{1}{2}$		$-\frac{1}{2} < d < 0$		$d < -\frac{1}{2}$		$d < 0$	
Time Saved via Freeway (mins.)	$t > 5$		$0 < t < 5$		$t < 2$		$0 < t < 2$		$t < 0$	
	User	Non User	User	Non User	User	Non User	User	Non User	User	Non User
Distance	124	31	30	14	92	135	57	256	55	763
Time	273	8	176	2	425	28	271	54	120	188
Orientation[a]	13	34	2	25	41	223	7	269	24	1150
Comfort and Convenience[b]	65	0	24	0	98	27	56	66	28	124
Miscellaneous[c]	7	18	8	3	16	28	19	46	12	167
TOTALS	484	91	240	44	672	441	510	691	239	2392

a) Orientation responses include signing, best way, habit, only way, scenic, unfamiliar

b) Comfort and Convenience responses include less traffic, no signals or stops, safer, "like it better"

c) Miscellaneous responses include cheaper, unknown.

Source: Reference 29

evident from the previous investigations that time alone does not accurately reflect the subjective value parameters. Cost was chosen since this one dimensional factor includes other values such as distance, safety and quality of traffic flow.

Resistance Measurement on the Route Component

Only one of the three possible measurements that may be used for the route component is the parameter R. There is no method to generate flow or pressure differential on this component. It has been postulated that the resistance function can be of two forms:

1. $R(y) = s(y) \cdot t(y)$

where: $s(y)$ = f(operating cost, accident cost, quality of flow cost) - a flow cost function in cents per vehicle mile.

$t(y)$ is a time flow function in hours per vehicle per link

2. $R(y) = S(y)$

where: $S(y)$ is a cost function in cents per vehicle mile = f(operating cost, accident cost, quality of flow cost, time cost).

This formulation requires that a relationship between travel time and volume be determined. Since travel time is the reciprocal of space mean speed, a relationship, for each link, between space mean speed and volume is required.

The relationships between speed, volume and density have been investigated for some time but due to the complexities of the flow phenomena, no single set of relationships can explain the variations

(15) (16) (19) (32). Each road section is probably unique in its combination of factors affecting flow.

The general relationship between the flow variables is as follows:

$$y(k) = k\ m(k)$$

where: y = volume or flow of vehicles per time unit

k = density or the number of vehicles per unit length

m = space mean speed or the mean speed of all the vehicles on a unit length of road at some instant.

Figure 11 schematically shows the generally accepted relationships between these variables. The schematic representation is shown since it is not known if the relationships are continuous and since these curves will vary with the type of road section, time of day, weather, population of drivers, etc.

Certain boundary conditions are evident from these diagrams. That is:

$y(0 \text{ density}) = 0;$

$y(k \text{ max.}) = 0$

$m(0 \text{ density}) = \text{mean free speed};$

$m(k \text{ max.}) = 0$

The boundary conditions for the speed-volume relationship are not so evident. However, it has been shown by many empirical studies that speed decreases as volume increases until some critical density is reached (15) (19). For any increase in density beyond this point, the relationship becomes unstable and speeds drop rapidly thus causing a further increase in density and a decrease in volume. Underwood (15) has postulated a speed-volume relationship that includes three zones.

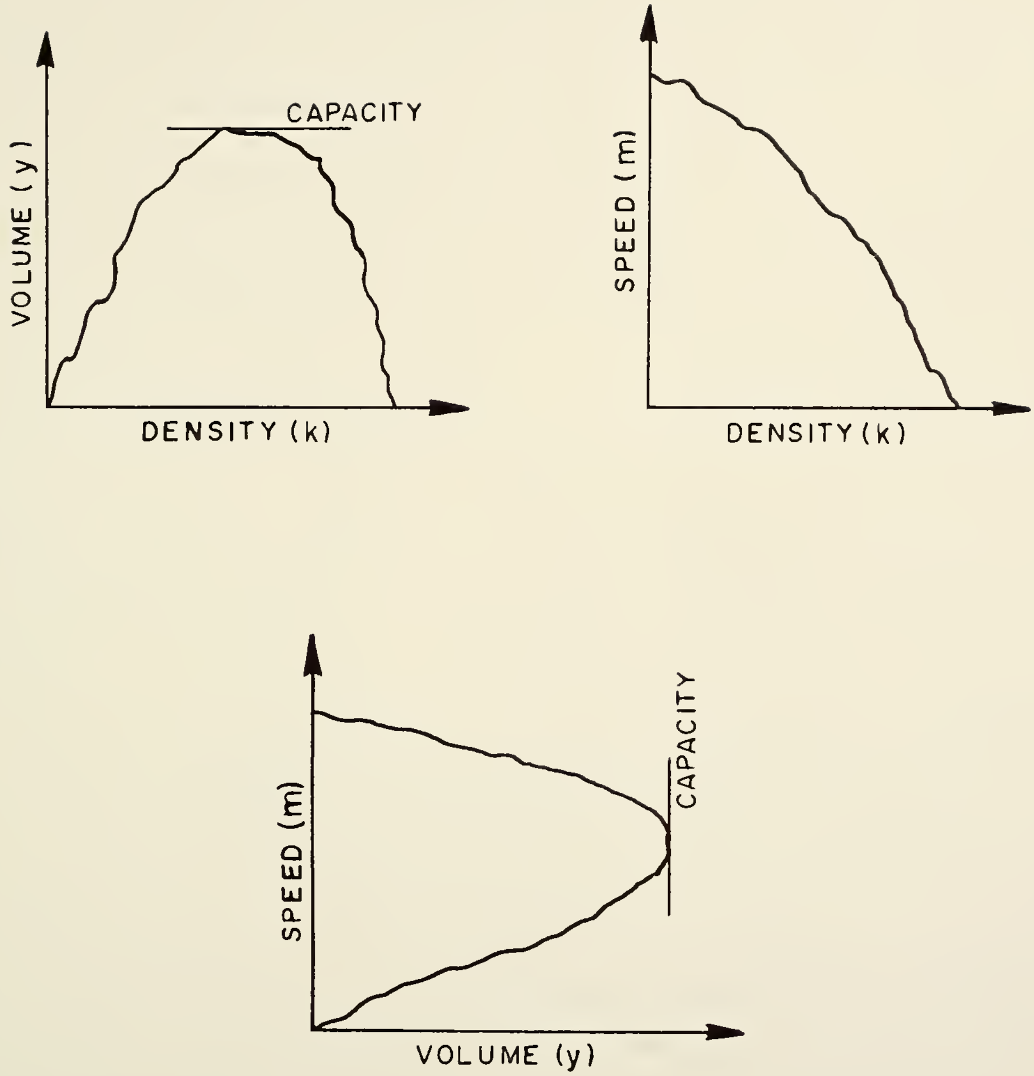


Fig. 11 FUNDAMENTAL DIAGRAMS OF ROAD TRAFFIC

One is a linear relationship between speed and volume up to some percentage of critical density. This was termed the zone of normal flow which would be a function of the roadway and other driving parameters. The zone of forced flow would follow a relationship as described by the lower curve in the Highway Capacity Manual (19), and be constant for all facilities. An intermediate zone of unstable flow would exist between normal and forced flow. No definite relationship between speed and volume would exist in this zone. This formulation has, in the writers opinion, much merit. However, in a time parameter traffic assignment, "demand" flow rather than actual flow is used. Hence, it is assumed that for any link, demand and actual flows should coincide up to some fraction of critical density. Beyond this point, only the relationship between demand flow and travel time need be considered since higher travel times are the results of queuing time.

Figures 12 and 13 show comparisons of speed-volume relationships used by the various network methods for freeway and arterial sections respectively. Both figures show a substantial difference between the travel functions used by the various methods.

As previously mentioned a time-flow relationship is required in the resistance function for each link in the system. This would involve a large number of equations. One method to reduce this number of relationships is to convert volume to a volume to capacity ratio. This provides a common basis for plotting relationships between the various road sections and yet accounts for the different loads and capacities.

Another technique to reduce the number of relationships

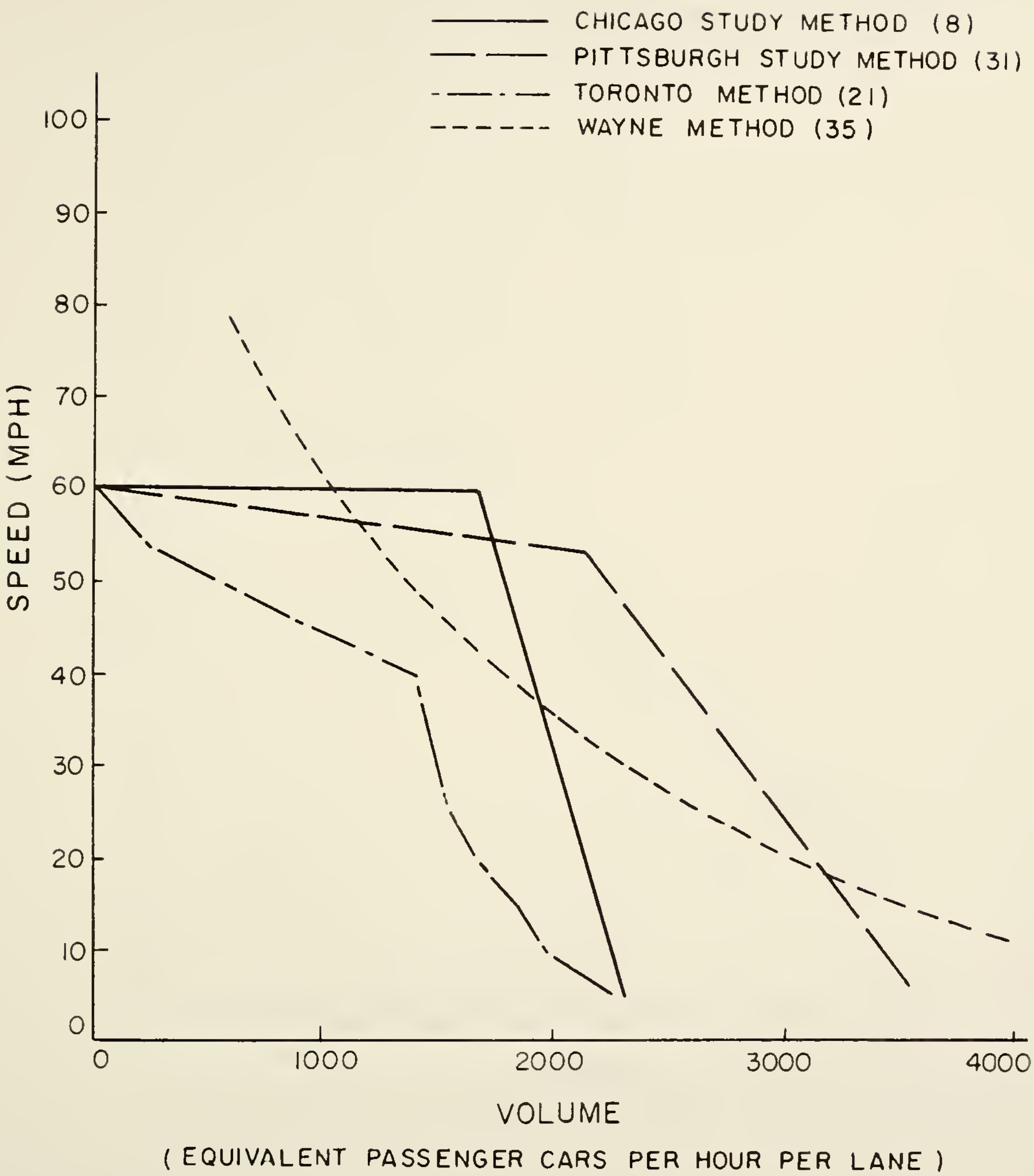


Fig.12 COMPARISON OF FREEWAY SPEED-VOLUME RELATIONSHIPS FOR EXISTING METHODS

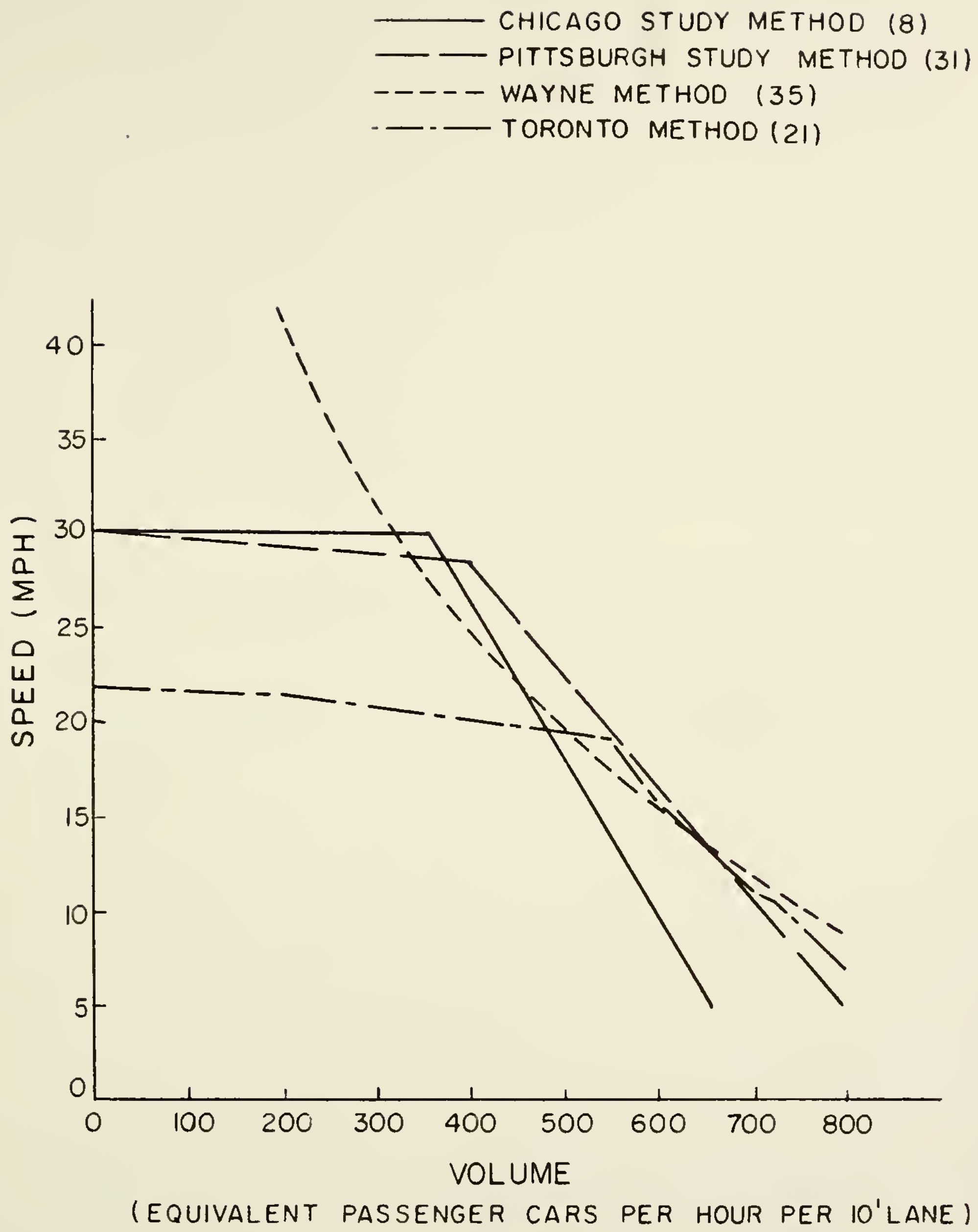


Fig. 13 COMPARISON OF ARTERIAL SPEED - VOLUME RELATIONSHIPS FOR EXISTING METHODS

required is to employ the delay concept as enunciated by Haikalis (17). This method established a relationship between the delay per vehicle, based on empirical studies, and volume to capacity ratio for a broad classifications of facilities (i.e. freeway, arterial). A free flow time (free speed) based on the type of facility and its location within the urban complex is established. The delay time based on volume is added to the free time regardless of the length of the link. Hence, a total time to traverse each link per vehicle is available. The average speed may then be computed.

Figure 14 shows the results of various empirical studies for freeways of speed vs volume and volume to capacity ratio. The capacity used in this analysis is the possible capacity in equivalent passenger cars per hour. (19)

Based on these relationships and a delay function proposed by Haikalis, a delay function was calculated which estimates actual flow conditions for freeways up to possible capacity and demand flow conditions beyond this point. That is:

$$d = 3.6 + \frac{7.5p}{1.2 - p} \qquad [1]$$

where: d is the average delay per vehicle mile in seconds

$p = \frac{v}{c}$ the volume to capacity ratio

v is the demand flow in equivalent passenger cars per hour

c is the possible capacity in equivalent passenger cars per hour.

This function is assumed to apply to all freeway links regardless of the free speed. A plot of the speed-volume relationship based on this delay function and a free speed of 55 m.p.h. is shown in

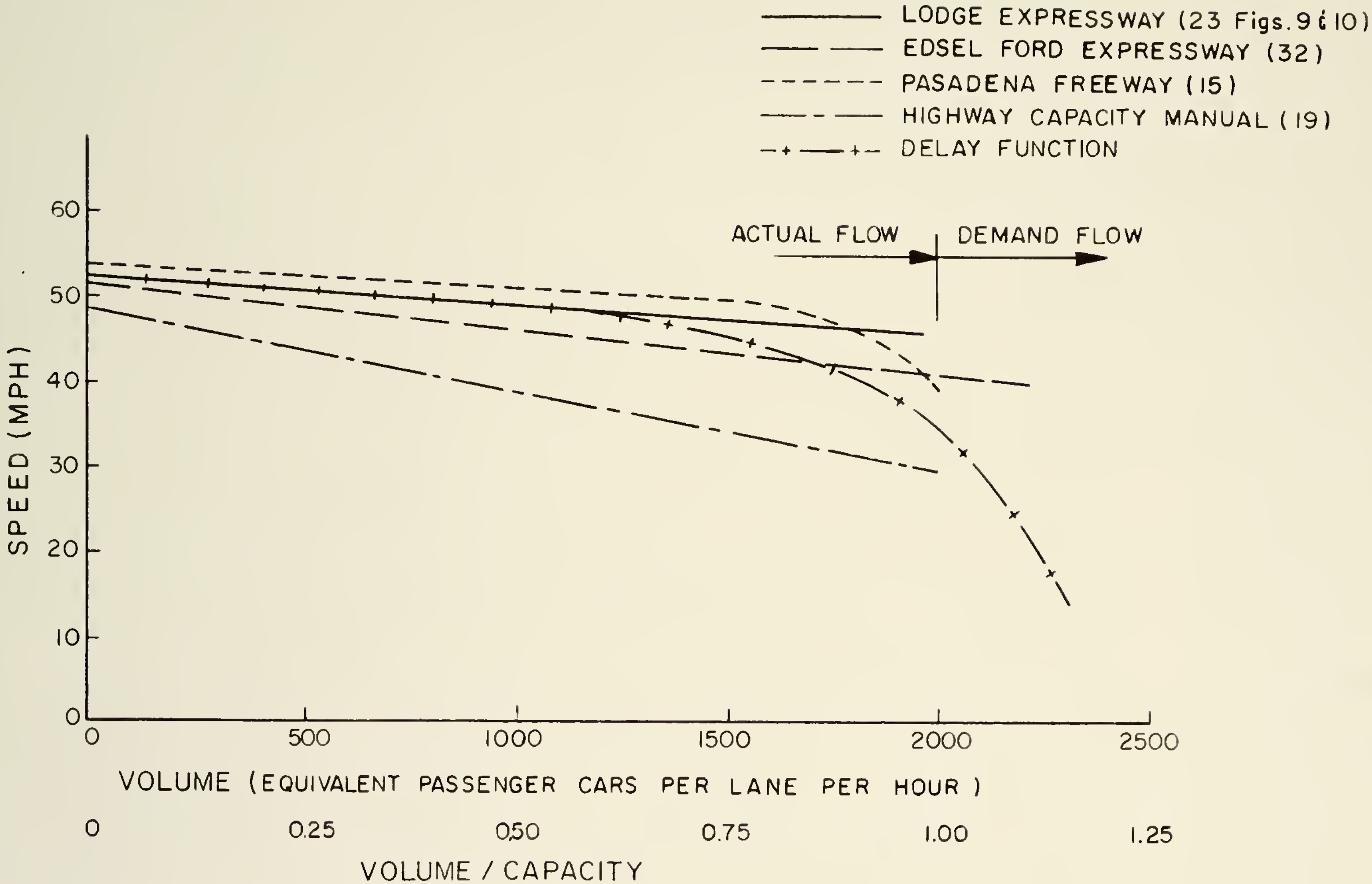


Fig.14 TYPICAL FREEWAY SPEED–VOLUME RELATIONSHIPS

Figure 14.

Unlike freeways, arterial streets are extremely diverse in their geometry, traffic control devices, etc. Because of this, little empirical information is available such that speed-volume relationships can be generated for each link.

One group of relationships which is in part based on empirical study has been shown under the discussion of the Traffic Research Corporation method (p.30). Another technique to establish these relationships was undertaken by Campbell et al (6). This latter technique assumed that all delay on an arterial street occurred at signalized intersections. Therefore, if the delays at these intersections could be measured in relation to volume, the travel time per link and hence the speed could be determined. The relationships generated by this study are shown in Figure 7. On the basis of this study, Haikalis developed the following arterial delay function (17).

$$d = 0.342\, e^{6.49p} \qquad .541 \leq p < 1.11$$

$$d = 11.5 \qquad 0 \leq p < 0.541$$

where: d is the average delay per vehicle in seconds per link

$p = {}^{y}/c$ volume to capacity ratio

c = maximum number of equivalent passenger cars per hour that can pass through an intersection approach if each signal cycle were fully loaded

y = volume in equivalent passenger cars per hour

Since this curve was not continuous, it was approximated by the relationship:

$$d = 7.5 + .093e^{7.5p} \qquad [2]$$

If the trip table ("through" drivers) are given in terms of hourly traffic flow, formulas [1] and [2] allow the calculation of travel time per vehicle for all links. As stated above, a "free speed" time is established for each link. To this is added the delay time and hence, total travel time per link can be determined.

However, if the trip table is given in terms of average daily traffic flows, then a conversion of formulas [1] and [2] to average daily delay functions is required.

The method of converting these hourly formulas to daily formulas requires a distribution of hourly traffic during the day and a relationship between hourly and daily capacity. An approximation of the hourly distribution of traffic is given by Haikalis as:

$$x = 0.1 - {}^{t}/200 \qquad 0 \le t \le 20$$

where x is the proportion of traffic occurring in the 't^{th}' highest hour.

It is interesting to note that the distribution used by Haikalis (Chicago) agrees quite closely to the mean distribution of hourly variations reported by Schuster (33). The hourly traffic flow can then be expressed as a proportion of daily traffic flow as:

$$y = xY$$

where: y is the hourly flow (equivalent passenger cars per hour)

Y is the daily flow (equivalent passenger cars per day)

The daily capacity is determined from the hourly capacity by assuming a constant peak hour. The usual relationship employed is based upon empirical evidence that peak hour flow is approximately 11 per cent of the daily flow with a 60 per cent split in the peak direction. Then

the hourly capacity is related to the daily capacity by:

$$c = 0.132\ C$$

where: c is the hourly capacity

C is the daily capacity

The hourly volume to capacity ratio, p, may be related to the daily capacity ratio, Z, by

$$p = \frac{v}{c} = \frac{xY}{.132C} = \frac{xZ}{.132}$$

The integration of the hourly delays, d, over all values of 't' produces a daily weighted average of the expected delay to each infinitesimal proportion of the daily flow.

$$D = \int_{t=0}^{t=20} \quad dxdt$$

where: D is the delay, seconds per vehicle, for daily flow.

The substitution of the relationships between x vs t, p vs Z and formula [illegible] for arterials permits the integration. The result is a functional relationship between D and Z for arterial streets.

$$D = 7.5 + \frac{.00577}{Z^{2}}\left[1 + e^{5.68Z}(5.68Z - 1)\right]$$

The above equation, because of its complexity, was approximated by:

$$D = 7.5 + .093\ e^{4.54Z} \qquad [3]$$

This equation effectively yields an average weighted proportion of traffic, x, in the 't' highest hour equal to .08.

The integrated expression for the average freeway daily delay, because of its complexity, was approximated by using the same weighted proportion as above, (i.e. $x = .08$). The resulting formula

for freeway daily delay is:

$$D = 3.6 + \frac{7.5\ Z}{1.98 - Z} \qquad [4]$$

The cost per vehicle on a link is the next function to be determined. The costs considered were those of operating, accident, quality of traffic flow and in one formulation time costs.

All of these costs are related to the speed of travel. Operating and accident costs related to speed have been determined (17). Quality of flow costs reflect discomforts of driving such as the number of speed changes required, lane changing, stop and go operation, etc. The establishment of these costs is quite subjective (15). Time costs have also been established but they are also quite subjective.

It is assumed that the discomfort costs vary uniformly from zero under optimum conditions (50 m.p.h.) to a maximum of 30 percent of the time costs as contained in Haikalis' report when the quality of flow is very poor (4 m.p.h.). Table 8 shows the derived relationship between speed and cost parameters.

The foregoing formulations permit the calculation of a resistance value for each link in a network in accordance with the postulated functions (page 63).

The postulated resistance function of the form

$$R(y) = s(y) \cdot t(y) \quad \text{(see page 63)}$$

was modified to

$$R(p, M) = K.Ms(p) \cdot t(p)$$

for hourly flows, and

$$R(Z, M) = K.Ms(Z) \cdot t(Z) \qquad [5]$$

for daily flows.

where: p = hourly volume to capacity ratio

Z = daily volume to capacity ratio

M = length of the link in miles

K = dimensional constant (assumed equal to 1.0)

$s(p)$, $s(Z)$ = cost function in cents per vehicle mile excluding time cost from Table 8

$t(p)$, $t(Z)$ = flow time in minutes per vehicle per link (arterials) or flow time in minutes per vehicle mile (freeways)

$= \frac{60M}{V_o}$ + appropriate delay function (formula [1], [2], [3] or [4])

V_o = free speed

The postulated resistance function of the form

$$R(y) = S(y) \quad \text{(see page 63)}$$

was modified to

$$R(p,M) = K.\ M.S(p)$$

for hourly flows, and

$$R(Z,M) = K.M.\ S(Z) \qquad [6]$$

for daily flows,

where:

$S(p)$, $S(Z)$ = cost function in cents per vehicle mile including time costs from Table 8.

TABLE 8

Cost Parameters Related to Speed

Cost in Cents per Vehicle Mile[a]

Average Speed (m)	Operating & Accident (b)	Quality (c)	Sub Total	Time (d)	Total
58	3.43	0	3.43	2.02	5.45
56	3.35	0	3.35	2.09	5.44
54	3.26	0	3.26	2.17	5.43
52	3.18	0	3.18	2.25	5.43
50	3.10	0	3.10	2.34	5.44
48	3.0[illegible]	.02	3.04	2.44	5.48
46	2.94	.05	2.99	2.54	5.53
44	2.86	.11	2.99	2.66	5.65
42	[illegible].78	.1[illegible]	2.90	2.79	5.69
40	2.70	.17	2.87	2.93	5.80
38	2.77	.22	2.99	3.08	6.07
36	2.86	.26	3.1[illegible]	3.25	6.37
34	2.97	.34	3.31	3.44	6.75
32	3.09	.40	3.49	3.66	7.15
30	3.21	.47	3.68	3.90	7.58
28	3.41	.54	3.95	4.18	8.13
26	3.61	.63	4.24	5.40	8.74
24	3.88	.78	4.66	4.38	9.54
22	4.19	.80	4.99	5.32	10.31
20	4.58	1.05	5.63	5.85	11.48
18	4.98	1.32	6.30	6.50	12.80
16	5.51	1.46	6.97	7.31	14.28
14	6.09	1.84	7.93	8.36	16.[illegible]9
12	6.79	2.24	8.03	9.75	18.78
10	7.94	2.81	10.75	11.70	22.45
8	9.32	3.66	12.98	14.63	27.61
6	10.75	5.07	15.82	19.50	35.3[illegible]
4	12.33	8.19	20.57	29.30	49.87

a) Equivalent passenger cars
b) Source reference (17)
c) Source reference (15) Cost $= (.30 - .\frac{03}{5}\, m)$(Time Cost)
d) Source reference (17)

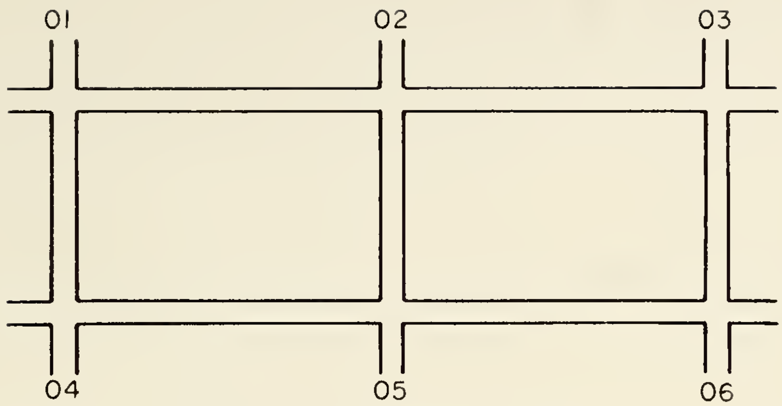
Systems Graphs

The systems graph is a set of component terminal graphs obtained by uniting the vertices of the terminal graph in a one to one correspondence with the components of the physical system. Figure 15 shows a hypothetical street system with trip table inputs along with the associated systems graph. This type of graph is not computationally efficient for large systems. A reduced graph may be obtained by summing the resistance values along the appropriate paths between a specified origin-destination pair.

The operations performed to obtain the reduced graph and solve the system are presented in the following section.

System Equations

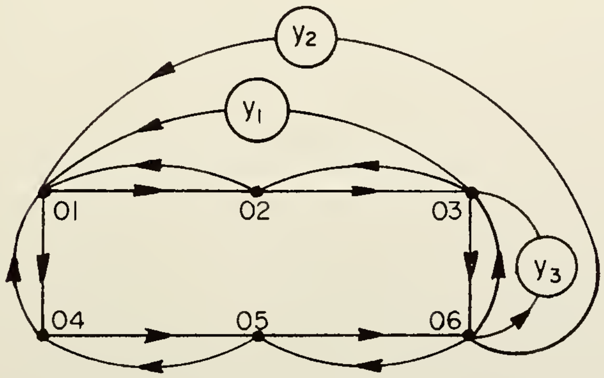
The operational procedures, presented below, to solve the assignment system are somewhat different than the techniques used in the solution of other physical systems by linear graph analysis. There are several reasons for these differences. In most physical systems the resistance value is a constant. However, in the assignment system the resistance value is a function of the unknown flow. The orientation of the non driver elements is arbitrary (i.e. negative flow is permissible) in most systems. The elements in the assignment system, since they correspond to the directional traffic flow, cannot change their orientation (i.e. negative flows are not permissible). Further, the normal graph analysis applied to the assignment system would consider all paths from an origin to a destination. This presents two difficulties. Firstly, the computation required to solve a large system would tax the largest computor. Secondly, the terminal equa-



TWO WAY STREET SYSTEM

TRIP TABLE

DESTINATION \ ORIGINS	01	03	06
01	x	x	x
03	y_1	x	x
06	y_2	y_3	x



SYSTEM GRAPH

Fig. 15 EXAMPLE OF SYSTEM GRAPH

tions of the components are not precise measurements as in other physical systems. This implies that not all paths between an origin-destination would be used by the motorist. For a large system this latter statement is intuitively appealing.

The operational procedure for the assignment system can be separated into two distinct parts. One, to find the appropriate paths. Two, to solve the sub-systems using linear graph analysis.

Path Determination

To find the "appropriate" path or paths between an origin-destination pair requires certain assumptions in any assignment method. The "minimum path" (with all or nothing assignment) is one such assumption. The limitations of this assumption have been discussed previously. In the capacity restraint type of solution, depending upon the restraint function used, it is possible to develop alternate paths which do not satisfy the evidence available from diversion studies. A more explicit assumption was formulated by Wardrop (37). This postulate states that the value function, X, between any origin-destination pair will be the same on all routes used and less than the value function, X, of even a single vehicle on any path between the same two points. Although this postulate is appealing, examples may be constructed such that it would be violated by the available evidence from diversion studies. In addition to the conceptual difficulties of this latter postulate, the calculations (and hence computor time) to find the "appropriate" paths are extremely time consuming.

To overcome these deficiencies, a path determination method which would be flexible, computationally efficient and satisfy the

diversion study evidence was sought. An algorithm was devised in an attempt to satisfy these objectives. In essence, this algorithm computes the "n" best paths in a network between an origin-destination pair subject to a diversion restraint. A repeated application of the algorithm to determine the best paths for all origin-destination pairs in a network is made.

The algorithm starts from a knowledge of all minimum resistance paths, based on free speeds, from all origins to all destinations. The minimum path algorithm employed was a modification of the Road Research Laboratory Algorithm (24) (39). The program for this algorithm is contained in Appendix C-1. For any origin-destination pair, the minimum path resistance value is multiplied by a diversion type factor. This factor is a variable in the program. Available evidence indicates a factor of approximately 1.3 would be appropriate for expressway diversion. Not too much evidence is available for the traffic splits between arterial routes. Hence, an appropriate factor here is somewhat indeterminate.

This product (diversion factor x minimum path resistance) will establish the maximum number of appropriate paths between any interzonal pair. The algorithm then systematically searches the network, using the previously determined minimum paths for all nodes, by "branch and stem" operations to find all paths whose resistance is less than the product value. An additional constraint is available in the program such that the number of allowable paths may be stated in advance. Thus if a purely diversion type of assignment is sought, the best two paths, subject to the diversion restraint, may be determined. In general, subject to the diversion restraint, the best "n" paths

between any origin-destination pair may be determined.

In more detail a path is traced out from the origin node until either:

- the destination zone is reached without exceeding the product value; or
- a "dead end" node is reached; or
- a path to the destination zone cannot be completed without exceeding the product value.

The last link of the path is then dropped, and the next link from the intermediate node is put in its place. The process is then repeated. If more than the specified number of paths are found, the path with the maximum resistance is deleted, and the new path is stored in its place.

The alternate paths for each origin-destination pair are kept in memory for the linear graph analysis.

Figure 16 shows the operational procedure or flow chart for this program. The program is contained in Appendix C-2.

This type of solution to the path determination problem, in the writer's opinion, satisfies the stated objectives. The program is flexible and relatively efficient from a computational point of view. It also overcomes the conceptual difficulties inherent in the existing techniques. That is, more than one path may be determined and the "demand" rather than the "restraint" paths are formulated. The paths determined by the algorithm are independent of assigned flow and are subject only to a diversion restraint.

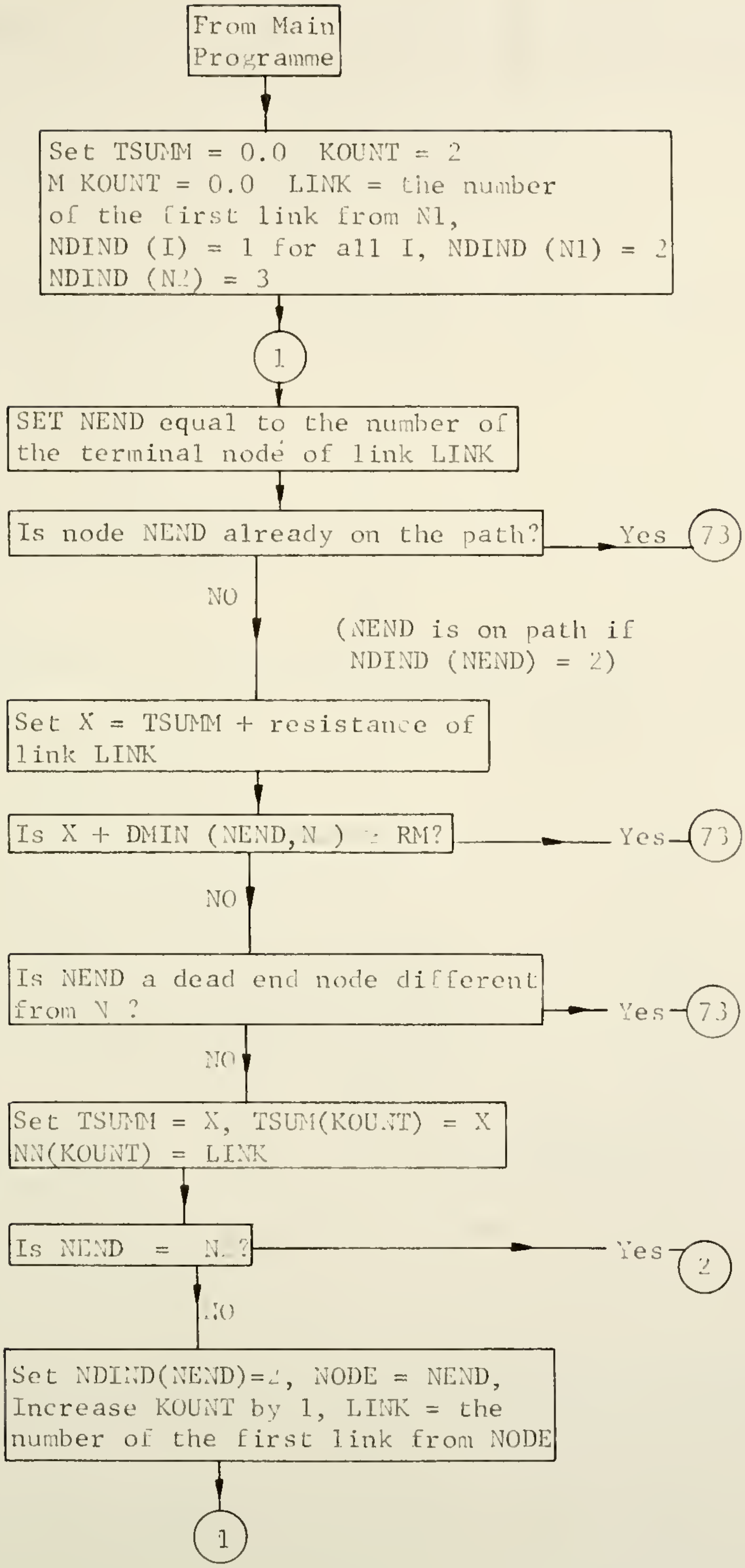
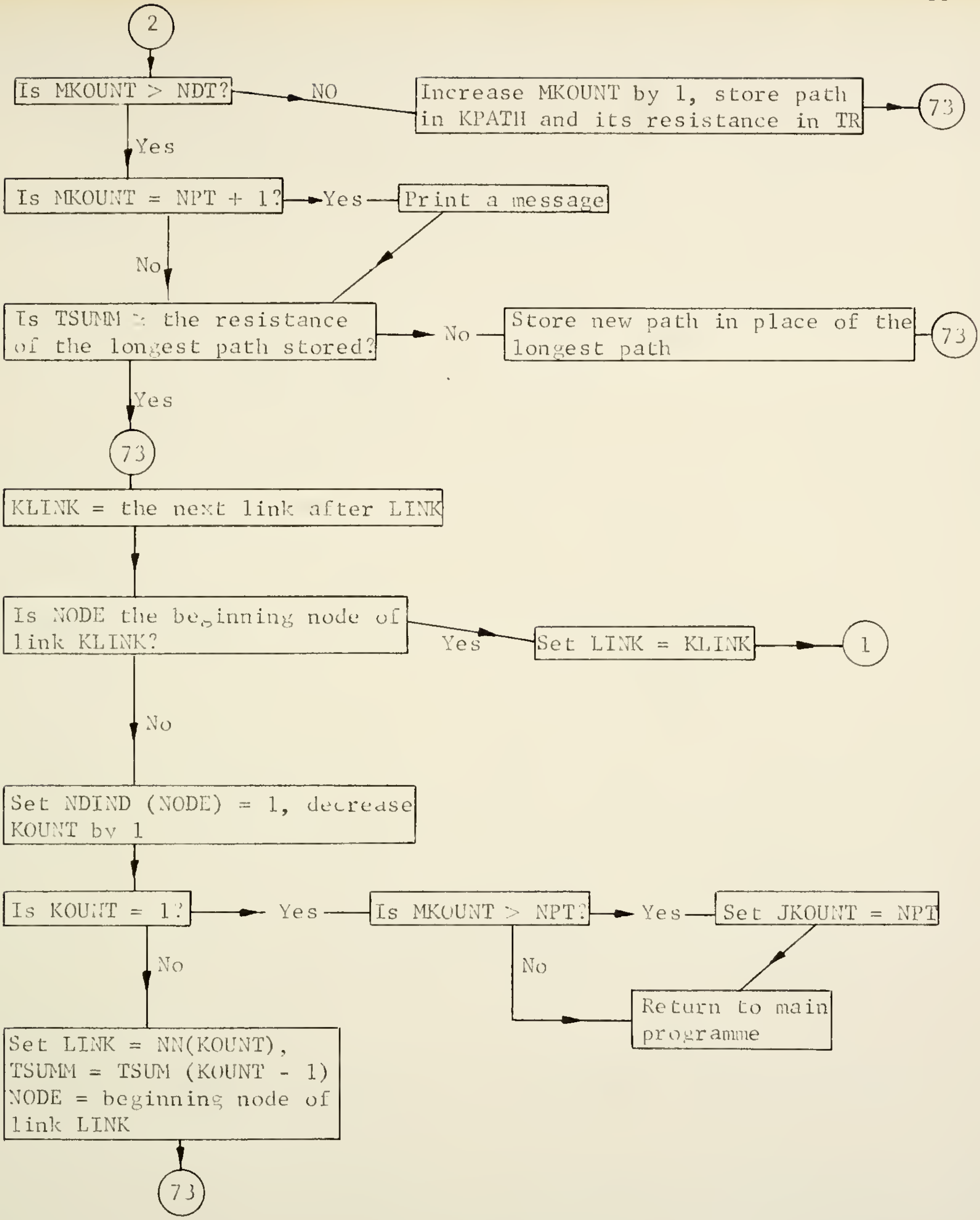


FIG. 16 FLOW CHART FOR PATH FINDING ROUTINE



For definition of terms see appendix B

FIG. 16 (continued)

Linear Graph Procedure

The operational analysis for the system solution, given the most likely paths between an origin-destination pair is as follows:

- a subgraph is established for each origin-destination pair. This subgraph consists of two vertices and as many elements as there are paths plus one driver element corresponding to the interzonal flow.

- the "demand" assignment is made using the path resistance values calculated in the path finding routine and solving for the path flows by the chord formulation. Assignment of these path flows to the appropriate links is then made. The process is repeated for each value in the trip table and the individual link volumes is accumulated.

For the "restraint" assignment, the previously calculated link loads are used to determine new link and path resistances. The same linear graph procedure is then repeated. Since the resistance values are flow dependent and non-linear, an iteration such that a balance between flow and resistance is required. This is achieved by averaging the flow values after each iteration. A unique solution is guaranteed if the resistance function is continuous and strictly increasing (10).

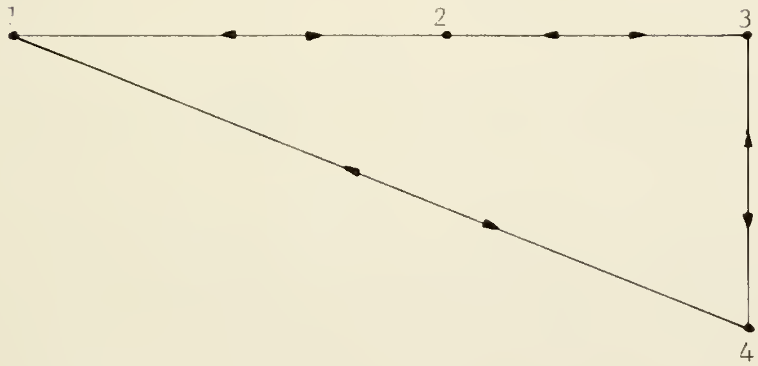
The computor program for this routine is contained in Appendix C-3.

A "long hand" solution of a small system is shown in the next chapter.

SOLUTION OF SYSTEMS

Example 1 - Synthetic System

A schematic of a two way street system is shown below. The vertices have been assigned mnemonics some of which are associated with the centroids of origin or destination zones.



The link descriptions and trip table are tabulated below:

No.	Link		Length (M)	Maximum Capacity	Free Speed
1	1	2	1	1200	40
2	1	4	2.0	4000	50
[illegible]	2	1	1	1200	40
4	2	3	0.5	1000	30
5	3	2	0.5	1000	30
6	3	4	0.5	1000	30
7	4	1	2.0	4000	50
8	4	3	0.5	1000	30

Trip Table (equivalent passenger cars per hour)

		Destinations			
		1	2	3	4
Origins	1	x	100	100	2500
	2	300	x	100	800

Path Determination

1. The resistance values were calculated using the postulated function:

$$R(p,M) = KM\ s(p)\ .\ t(p)$$ (page 63)

and the delay functions:

$$d = 5.6 + \frac{7.5p}{1.\text{[illegible]} - p}$$ (page 69 freeway)

$$d = 7.5 + .093e^{7.5p}$$ (page 71 arterials)

An example calculation for the zero flow condition on link number 1 follows:

$$t(p) = \left(\frac{60M}{V_o} + \frac{d}{60} \right) \frac{1}{M}$$

where: t = travel time (minutes per mile)

V_o = free speed (m.p.h.)

d = appropriate delay function (seconds per mile)

M = length of the link (miles)

$$t(p = 0) = \frac{60}{40} + \frac{7.5 + .093e^{0}}{60} = 1.627 \text{ minutes per mile}$$

Speed = 36.9 m.p.h.

From Table 8, the cost (exclusive of time) is:

$s(p = 0)$ = 3.06 cents per vehicle mile

Therefore, the resistance value is:

$$R(\rho = 0, M = 1) = 1 \times 1 \times 3.06 \times 1.627 = 4.90$$

2. Minimum path trees were determined for all origins to all destinations at resistance values corresponding to zero flow conditions. These paths are recorded in Table 9.

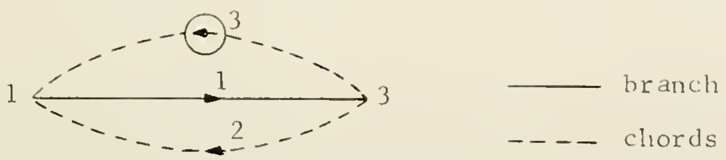
3. A diversion factor of 1.3 was used to multiply each minimum path resistance value. Paths whose resistance values are less than the product value from each origin-destination pair were found. These are recorded in Table 9.

Linear Graph Procedures

1. For those origin-destination combinations which have only one path, the trip table inputs are assigned.

2. Subgraphs are formed from the remaining trip table inputs and solved by the chord formulation.

An example is shown below for the subgraph of origin 1 to destination 3.



element 1 - path 1, 2, 3

element 2 - path 1, 4, 3

element 3 - trip table input

TABLE 9

PATH DETERMINATION - Example 1

Origin	Destination	Minimum Path (a)	Diversion PATH (a)	Minimum Path (b)	Diversion PATH (b)
1	2	1, 2	-	1, 2	-
	3	1, 2, 3	1, 4, 3	1, 2, 3	-
	4	1, 4	-	1, 4	1,2,3,4
2	1	2, 1	-	2, 1	-
	3	2, 3	-	2, 3	-
	4	2, 3, 4	-	2, 3, 4	-

a) Based on resistance function $R(p,M) = K.M.s(p) \,.\, t(p)$

b) Based on resistance function $R(p,M) = K.M.S(p)$

The circuit equations may be represented in general form as:

$$\begin{bmatrix} B_{11} & B_{12} & U & 0 \\ B_{21} & B_{22} & 0 & U \end{bmatrix} \begin{bmatrix} X_{b-1} \\ X_{b-2} \\ X_{c-1} \\ X_{c-2} \end{bmatrix} = 0$$

The first term, X_{b-1} is non-existent in this system, hence the circuit equations are:

$$\begin{bmatrix} B_{12} & U & 0 \\ B_{22} & 0 & U \end{bmatrix} \begin{bmatrix} X_{b-2} \\ X_{c-1} \\ X_{c-2} \end{bmatrix} = 0$$

The terminal equations of the street components may be represented as:

$$\begin{bmatrix} X_{b-2} \\ X_{c-1} \end{bmatrix} = \begin{bmatrix} R_{b-2} & 0 \\ 0 & R_{c-1} \end{bmatrix} \begin{bmatrix} Y_{b-2} \\ Y_{c-1} \end{bmatrix}$$

where: R_{b-2} = the sum of the link resistances corresponding to branch paths

R_{c-1} = the sum of the link resistances corresponding to chord paths

Y_{b-2} = flow on the branch paths

Y_{c-1} = flow on the chord paths

Specifically for the demand assignment:

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 9.51 & 0 \\ 0 & 12.17 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

The next sequence is the substitution of the subgraph funda-

mental circuit equations into the chord formulation set of equations (see Chapter 3).

$$\begin{bmatrix} 0 \\ \\ u \end{bmatrix} X_{c-2} + \begin{bmatrix} B_{12} & U \\ \\ B_{22} & 0 \end{bmatrix} \begin{bmatrix} R_{b-2} & 0 \\ \\ 0 & R_{c-1} \end{bmatrix} \begin{bmatrix} B_{12}^T & B_{22}^T \\ \\ u & 0 \end{bmatrix} \begin{bmatrix} y_{c-1} \\ \\ y_{c-2} \end{bmatrix} = 0$$

where: B_{12} is a column matrix with coefficients equal to -1. The number of rows of this matrix correspond to the number of non-driver chords in the subgraph; or it corresponds to the number of paths less one between an origin-destination pair

B_{22} is +1 corresponding to the driver or the trip table input

y_{c-1} is the unknown flows for the non-driver chord elements.

y_{c-2} is the through driver (the trip table input)

For the example, the specific formulation is:

$$\begin{bmatrix} 0 \\ \\ 1 \end{bmatrix} X_3 + \begin{bmatrix} -1 & 1 \\ \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 9.51 & 0 \\ \\ 0 & 12.17 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ \\ 1 & 0 \end{bmatrix} \begin{bmatrix} y_2 \\ \\ 100 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 \\ \\ 1 \end{bmatrix} X_3 + \begin{bmatrix} 21.68 & -9.51 \\ \\ -9.51 & 9.51 \end{bmatrix} \begin{bmatrix} y_2 \\ \\ 100 \end{bmatrix} = 0$$

Taking the first set of the above equations the solution is:

$y_2 = 44$ v.p.h.

The flow on element 1 is solved by subtraction.

$y_1 = 100 - 44 = 56$ v.p.h.

The results of the total demand assignments are shown as Y_1 in Table 10.

3. For the capacity restraint assignment, new link and path resistance values are calculated corresponding to the flows obtained from the demand assignment. The linear graph routine is employed again to calculate the restrained volumes. If these values are within "tolerable" limits of the demand volumes, the restraint assignment is complete. If the values are not within "tolerable" limits an iterative solution is required. The results of the first restraint solution are shown as Y_2 in Table 10.

4. The iterative solution (not required in this example) is the process of balancing link volumes and resistances. This is achieved by averaging the link volumes according to:

$$\bar{y} = \sum_{i=1}^{n} \frac{y_i}{n}$$

where: $\bar{y}$ = average assigned volumes

y_i = trips assigned to the links during the i^{th} iteration of the linear graph routine (including the demand assignment)

n = the number of linear graph iterations,

and repeating the linear graph routine.

The same example was used to find the assigned volumes using the postulated resistance function:

$$R(p,M) = K.M.S(p) \qquad \text{(see page 63)}$$

The results of the demand assignment and restraint assignment are shown as Y_3 and Y_4 respectively in Table 10. The paths developed using this function are shown in Table 9.

The postulated product resistance function yields different paths than the straight cost resistance function. The former function

TABLE 10

LINK VOLUMES - Example 1

No.	Link	R_o	$Y_1^{(a)}$	R_1	$Y_2^{(b)}$	$Y_3^{(c)}$	$Y_4^{(d)}$
1	12	4.90	156	4.98	138	1290	625
2	14	7.56	2544	8.42	2562	1410	2075
3	21	4.90	300	5.00	300	300	300
4	23	4.61	956	16.40	938	2090	1425
5	32	4.61	0	4.61	0	0	0
6	34	4.61	800	5.80	800	1890	1225
7	41	7.56	0	7.56	0	0	0
8	43	4.61	44	4.61	62	0	0

a) Demand link flows based on the product resistance function

b) Restrained link flows based on the product resistance function

c) Demand link flows based on the straight cost resistance function

d) Restrained link flows based on the straight cost resistance function

favours expressway usage. The product function, in this example, yielded assigned volumes close to the volumes obtained using a time ratio diversion assignment. The straight cost function assigned volumes which approached the California diversion assignment.

Example 2 - Synthetic System

This example has been arbitrarily chosen to further evaluate the linear graph assignment algorithm by comparison with the existing techniques.

A schematic of the system together with the link descriptions is shown in Figure 17.

The trip table is shown below (entries are equivalent passenger cars per hour).

		Destinations										
		1	2	3	4	5	6	11	12	13	15	15
	10	200	200	200	300	400	2000	200	200	300	300	400
Origins	15	x	x	x	x	x	200	100	100	200	400	x

Linear Graph Assignment

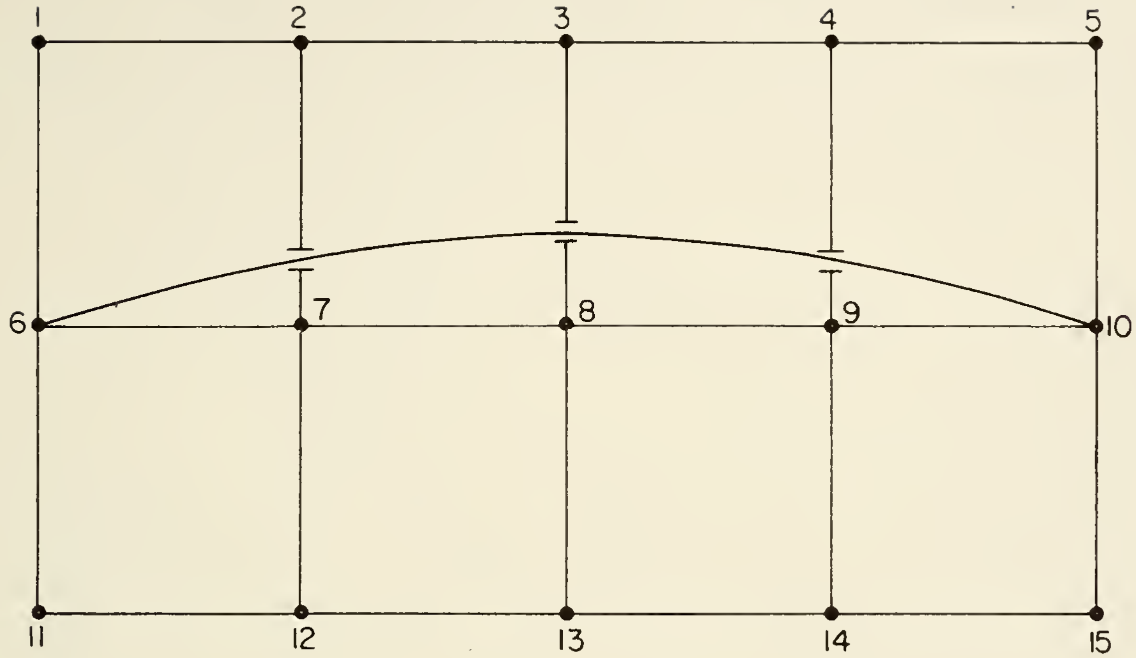
Assignments were made to this system utilizing the I.B.M. 7040 computor of the University of Waterloo.

A. The results of the path determination routine are shown in Table 11 for the postulated resistance function of the form:

$$R(p,M) = K.M.\ s(p).\ t(p)$$

The link volumes based on the above function are shown in Table 12 (Y_1)

B. The path determination routine for the postulated resistance



ALL STREETS TWO DIRECTIONAL
ALL LINKS EXCEPT 610 AND 106 : 0.5 MILE
LINKS 610 AND 106 : 2 MILES

0.5 MILE LINKS : FREE SPEED 30 M.P.H. CAPACITY =1200 V.P.H.
2.0 MILE LINKS : FREE SPEED 50 M.P.H. CAPACITY = 4000 V.P.H.

Fig. 17 SCHEMATIC OF SYSTEM - EXAMPLE No 2

TABLE 11

Paths Developed by Various Network Methods - Example 2

Origin	Destination	Path 1	Path 2	Path 3	Path 4	Path 5	Path 6
10	1	10,6,1*	10,9,4,3, 2,1(b)	x	x	x	x
	2	10,5,4,3, 2*	10,9,4,3, 2**	10,9,8,7, 2+(b)	10,6,7, 2+(b)	10,9,8,3, 2+(b)	10,6,1,2+
	3	10,5,4,3*	10,9,4,3**	10,9,8,3+(a)	x	x	x
	4	10,5,4*	10,9,4**	x	x	x	x
	5	10,5*	x	x	x	x	x
	6	10,6*	10,9,8,7,6(b)	x	x	x	x
	11	10,6,11*	10,9,8,7,12, 11(b)	x	x	x	x
	12	10,9,8,7, 12*	10,6,7,12**	10,9,8,13, 12**	10,15,14, 13,12+(b)	10,6,11, 12+(b)	10,9,14,13, 12+ (b)
	13	10,9,8,13*	10,9,14, 13**	10,15,14, 13+(b)	x	x	x
	14	10,9,14*	10,15,14**	x	x	x	x
	15	10,15*	x	x	x	x	x
15	6	15,10,6*	15,14,13,12, 11,6(b)	x	x	x	x
	11	15,14,13, 12,11*	15,10,6,11**	x	x	x	x
	12	15,14,13,12*	x	x	x	x	x
	13	15,14,13*	x	x	x	x	x
	14	15,14*	x	x	x	x	x

TABLE 11 (contd.)

* all methods

** all methods except Chicago and Pittsburgh

+ Linear Graph Methods

(a) Wayne Method

(b) Traffic Research Corporation Method

TABLE 12

Assignment by Network Methods - Example [illegible]

Link	Y_1	Y_2	Y_3	Y_4	Y_5
12	37	31	0	0	0
21	0	0	0	0	70
32	96	104	200	125	200
43	200	204	400	275	400
54	267	261	700	410	350
61	237	231	200	200	130
67	76	64	0	125	120
611	303	283	200	225	130
72	68	65	0	75	70
76	0	0	0	0	600
712	73	68	200	175	145
83	95	99	0	50	0
87	64	69	200	125	695
813	150	143	300	250	115
94	234	243	0	165	350
98	309	312	500	425	810
914	338	311	300	300	255
105	667	661	1100	810	750
106	2815	2779	2600	2750	2000
109	881	866	800	890	1415
1015	600	646	400	465	705
116	0	0	0	0	80
1112	39	32	0	0	50
1211	37	48	100	75	200
1312	225	248	200	200	305
1413	575	605	400	450	690
1510	263	252	200	215	170
1514	937	994	800	850	1135

Y_1 - Linear Graph Method: R = K.M.s(p). t(p)

Y_2 - Linear Graph Method: R = K.M. S(p)

Y_3 - Chicago & Pittsburgh Method

Y_4 - Wayne Method

Y_5 - Traffic Research Corporation Method

function of the form:

$$R(p,M) = K.M.S(p)$$

are shown in Table 11. The link volumes for this function are shown in Table 12 (Y_2).

In this example, the postulated resistance functions yielded the same paths from all origins to all destinations. The diversion factor employed was 1.3. The assigned volumes between the two postulated functions did not differ materially. However, the product resistance function is sensitive to volume changes and becomes quite large at flows between practical and possible capacities for arterials. Freeway resistances remain relatively low even at high volumes. This is reflected in the higher assignment to the freeway link.

Diversion Assignment

The diversion assignments were based on mean operating speeds of 44 m.p.h. and 22 m.p.h. for the freeway and arterial links respectively. These speeds were based upon the possible link loadings achieved by the linear graph method and the delay functions previously developed. Diversion assignments are usually made to freeways. Hence, only the assignments to the freeway section of the example are shown in Table 13.

The variability in diversion assignments is evident from these results. The range of values assigned to the freeway was 900 equivalent passenger cars per hour; which is the difference between the time ratio and distance ratio methods. The linear graph algorithm assignment using either of the postulated functions closely approximated the time ratio and Detroit diversion assignments to the freeway link.

TABLE 13

Diversion Assignments - Example 2

Method	(a) Volume Link 106	Source
Time Ratio	2810	Figure 1
Distance Ratio	1910	Figure 3
Detroit	2750	Figure 5
California	2170	Figure 6

a) Equivalent Passenger Cars per hour

Chicago and Pittsburgh Network Methods

Because of the relatively low trip table volumes, these methods would yield the same results provided loading node 15 was the first tree building node selected. The particular example selected yields duplicate minimum paths to some destination nodes under these methods of assignment. This was arbitrarily overcome by loading the tree from origin zone 10 to destination zones 12, 13, 14 and 15, then building another tree to the remainder of the destination zones. The assigned volumes are shown in Table 12 (Y_3). It will be noted that fewer links are assigned volumes by these methods than by the other network methods. Path 1 of Table 11 covers all of the alternate paths developed by these methods.

Wayne Assignment Method

The results of assignment by this method are shown in Table 12 (Y_4). Paths developed by this method are shown in Table 11. The assumptions made in the application of this method were as follows:

- Practical capacity arterial links 800 vehicles per hour
- Practical capacity freeway links 1500 vehicles per hour
- travel time at practical capacity arterials - 1.5 minutes
- travel time at practical capacity freeways - 3.0 minutes

Nine iterations were required to achieve a reasonable balance between suceeding average flows.

This method assigns traffic to the various paths between any origin-destination pair such that if enough iterations were carried out, the travel times between these paths would be equal and less than the travel time of a single vehicle on any other path. The differences

between this method and the graph method are in the path determination and iteration techniques. The graph method includes paths which follow a "diversion" type route. Further, the linear graph solution ensures equal values, X, between all routes of an origin destination pair without prolonged iteration.

Traffic Research Corporation Method

The paths used in this method are shown in Table 11. The results of the assignment are summarized in Table 12 (Y_5). The capacity functions as employed by this method are more flow sensitive for freeway travel than the other methods. Hence, the volume assigned to the freeway under this method is the lowest of all the network methods investigated. Further, certain arterial links develop assignments greater than the possible capacities.

Discussion of Results

The variability of the assigned volumes is evident from Tables 12 and 13. As previously mentioned, for this example, the linear graph algorithm showed similar assigned link volumes under either of the postulated resistance functions. This would not be true if a large number of links were assigned volumes between practical and possible capacities. The product resistance function under these conditions would assign more traffic to freeway links. Examination of Table 11 shows one of the primary differences in this method of assignment from the other multi-path network methods. The linear graph algorithm develops its paths independently of assigned volumes. In this example, more paths were developed by this algorithm than by the other methods. An examination of the network (Figure 17) for paths between

origin node 10 and destination node 2 will be used to illustrate this difference. The Wayne method only developed two paths between these zones; neither of them utilizing the freeway link. All diversion methods would assign volumes to this link. The Wayne method only utilizes two paths to this destination, whereas the topology indicates two other paths whose resistance are equal to that of the paths selected. Nine iterations were required for reasonable closure in the Wayne method whereas only one restraint assignment was required by the linear graph algorithm.

The Traffic Research Corporation method developed almost as many paths as the graph algorithm. However, certain paths of equal resistance values were not developed. Seven iterations were carried out for the solution of this system. Oscillation of assigned volumes occurred between iterations indicating the closure problems in this method. The averaging technique employed in the graph method was used to speed closure.

The example was not too well suited to the one path methods of assignment. However, it does indicate one major weakness in the method. That is, the problems that occur when two or more paths of equal or near equal resistances occur. Only one of these paths may be selected.

Example 3 - Brockville Ontario

To further evaluate the proposed assignment technique a "real" city was chosen and the assigned link volumes were compared to existing ground counts. The city of Brockville, Ontario was chosen for this purpose since the data for this city was readily available. The trans-

portation study for this city was conducted by M.M. Dillon, Consulting Engineers, Toronto, in 1963 under the auspices of the city of Brockville and the Ontario Department of Highways. In October 1964 a report of this study, "Brockville Area Transportation Study", was published. The city of Brockville is situated on the St. Laurence River between Montreal and Toronto. It has an area population approaching 20,000.

The assignment of existing trips to an existing network is the only means of evaluating the adequacy of the traffic assignment technique. The accuracy of the assignment is best determined by a link by link comparison with ground counts. Screenline checks may also indicate the accuracy of the assignment but depending upon the type of screenline it is only a gross check. These comparisons, however, only measure the total error and not the error attributable to the traffic assignment procedure. The sources of error are composed of the following:

1) errors in the trip survey and expansion (i.e. trip table)

2) errors in the ground counts and expansion

3) errors in the assignment procedure

An external-internal origin and destination survey was conducted for the city of Brockville. The internal survey was made by a 20% sample of motor vehicles registered in Brockville and telephoning the owners to establish the sample two hour (4.00 p.m. to 6.00 p.m) trip distribution. The survey data was then expanded to form the average 4.00 p.m. to 6.00 p.m. trip distribution. The error commonly attributed to this phase of the survey is determined by screen line checks. In Brockville this error amounted to eleven percent. However,

this error is confounded with the possible ground count errors.

The errors in the assignment procedure can be attributed to many sources; a course network (i.e. too few links in the network), the zone sizes are too large, failure to assign intrazonal trips, faulty speed and delay information, faulty value functions, assumptions of assignment model.

There appears to be no way, with the data that is available, to evaluate the portion of the total error that is attributable to the above factors. Nevertheless, as previously mentioned, the comparison of assigned volumes to ground counts is the only means of evaluating the technique and is an indication of its accuracy.

The zone map used for the internal and external origin destination survey is shown in Figure 18. The street classification is shown in Figure 19. A schematic of the network whose vertices have been assigned mnemonics, some of which are associated with the centroids of origin or destination zones, is shown in Figure 20. Because of the street configuration and trip table certain zones have been combined in the analysis.

The input data for the system, in addition to the network topology, consists of the trip table (Table 14), the link data (Table 15) and the cost table (Table 8). For assignment to an existing network the use of the delay function was not employed since operating speeds were available from the study. With the operating speeds known, Table 8 may be used directly to find the appropriate resistance values. The delay functions formulated in this report are designed to predict the operating speeds for future facilities.

The free speeds shown in Table 15 were estimated from inform-

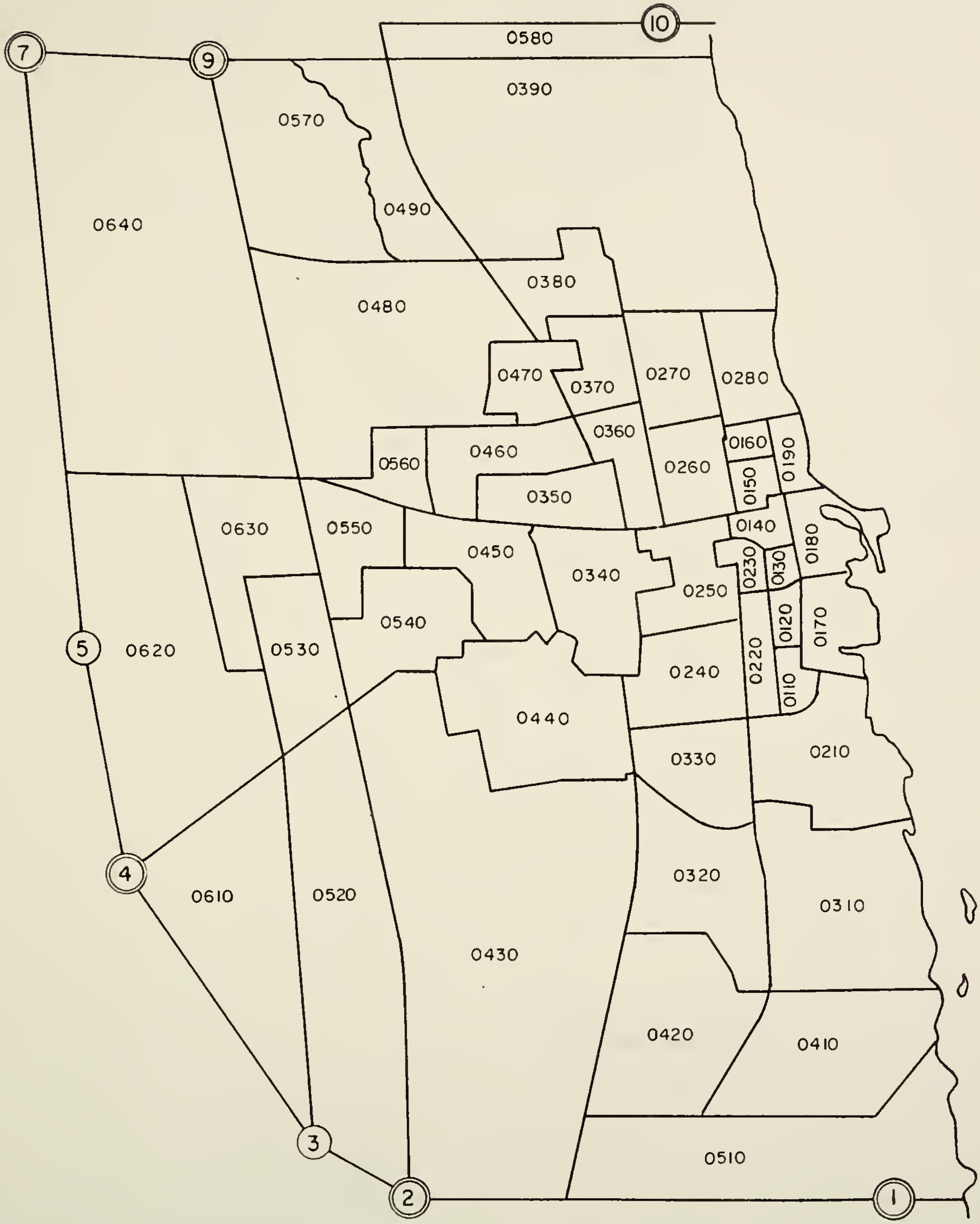


Fig.18 ORIGIN AND DESTINATION ZONES
BROCKVILLE , ONTARIO

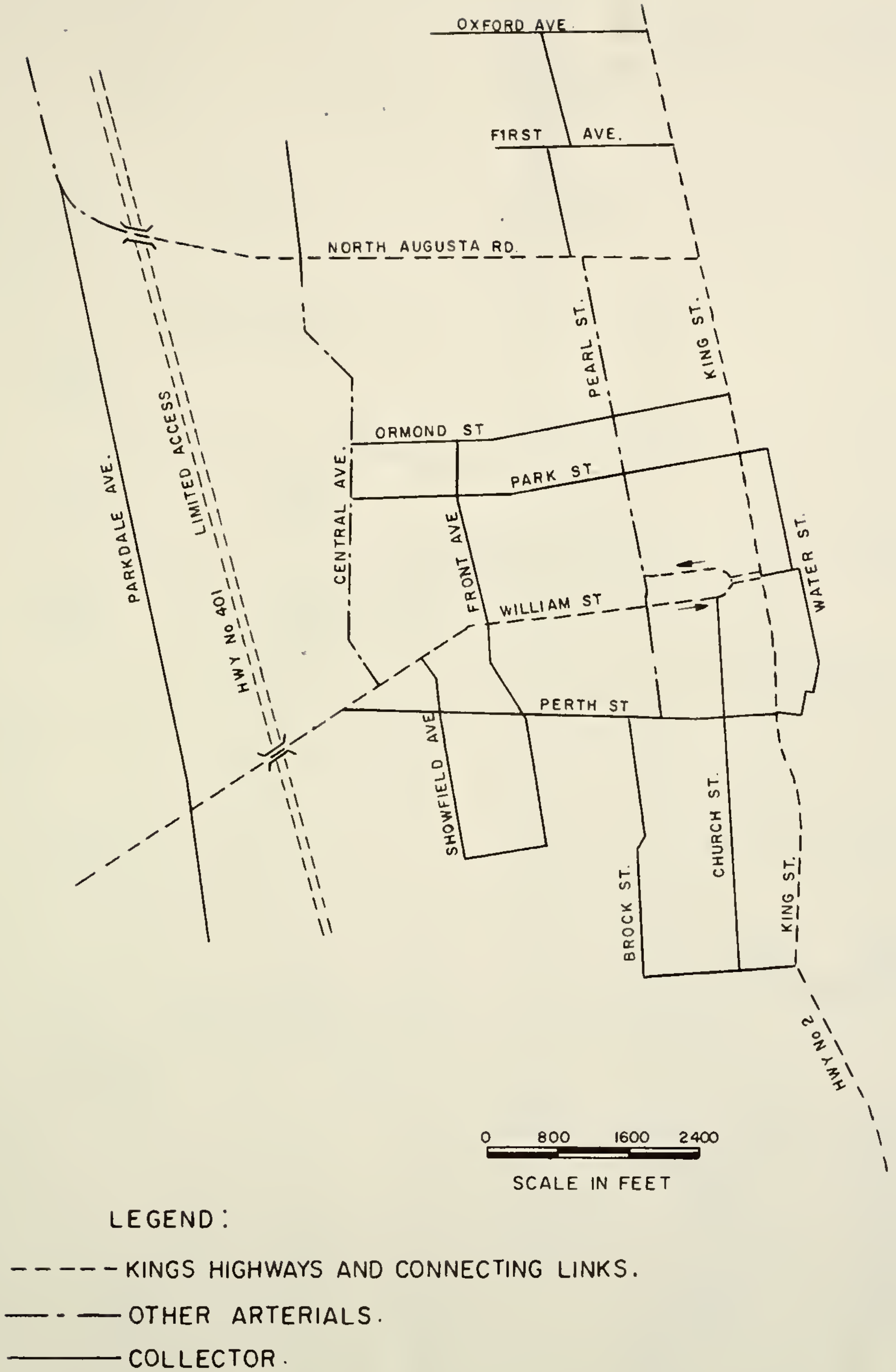


Fig. 19 STREET CLASSIFICATION
BROCKVILLE, ONTARIO

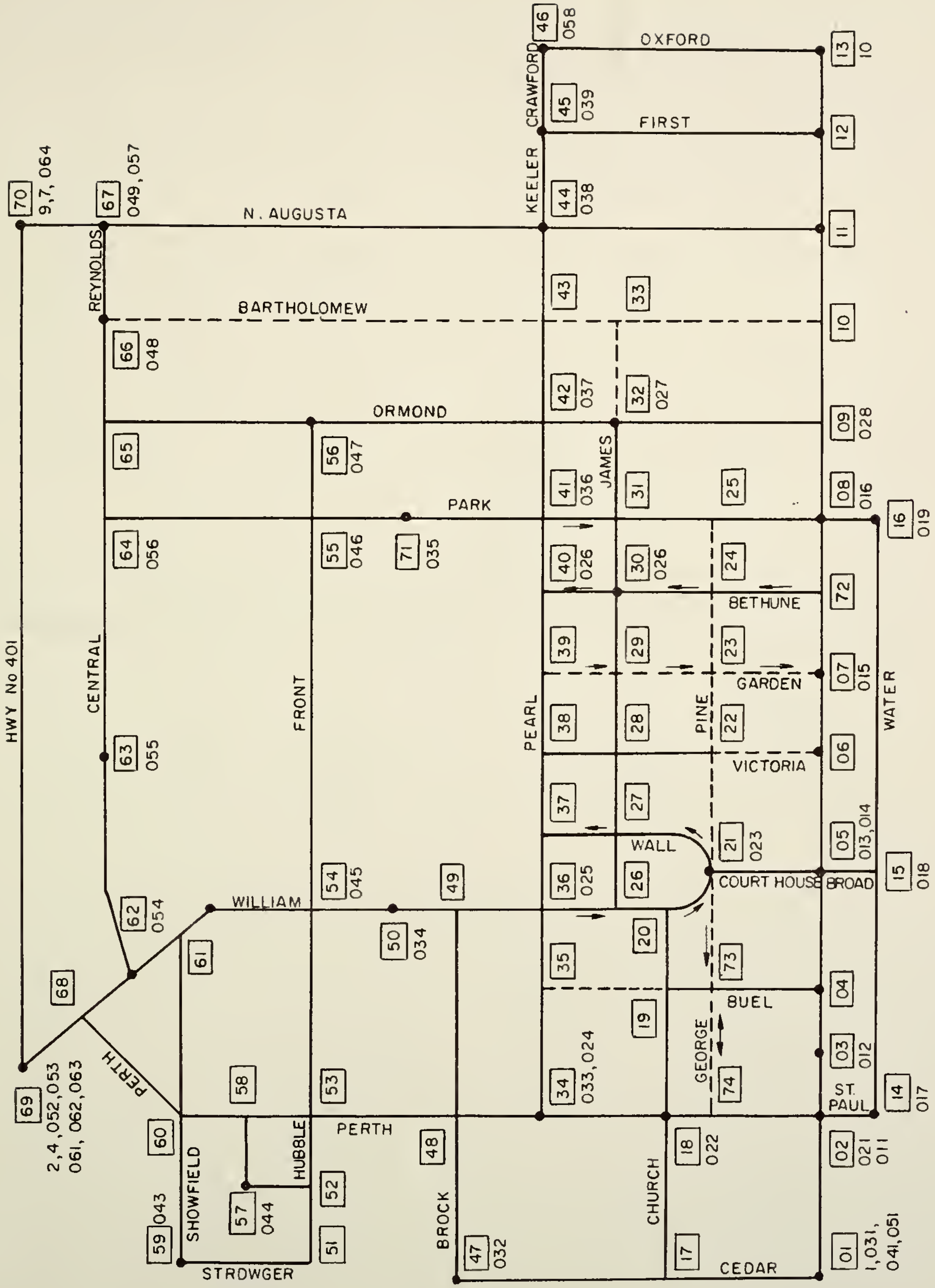


Fig. 20 NETWORK MAP
BROCKVILLE, ONTARIO

TABLE 14 - TRIP TABLE

PEAK PERIOD - 4.00 to 6.00 p.m. Average Weekday

Destinations

Origins	01	02	03	05	07	08	09	13	14	15	16	18	21	30	32	34	36	41	42	44
01	x	30	39	116	11	2	3	31	11	19	17	15	4	18	10	47	20	0	3	16
02	42	x	14	37	7	0	0	10	7	20	0	15	0	8	21	0	0	0	0	0
03	4	21	x	42	0	0	23	5	7	7	0	0	7	22	12	31	16	0	0	5
05	112	40	23	x	5	25	21	34	33	7	7	0	8	18	21	43	21	14	0	16
07	41	0	16	11	x	0	7	14	0	7	0	0	0	7	7	7	8	0	0	0
08	2	7	0	20	14	x	0	3	7	0	0	0	0	0	0	7	7	0	0	0
09	4	7	0	0	0	0	x	5	14	0	0	0	0	8	8	7	8	7	0	0
13	41	15	4	39	13	8	11	x	5	11	3	7	3	12	16	13	15	8	9	7
14	27	43	8	20	0	7	7	7	x	0	0	0	0	0	0	4	7	0	0	0
15	18	20	4	12	0	7	0	11	7	x	7	0	8	0	0	21	0	14	0	0
16	5	0	0	4	0	0	0	2	0	7	x	0	0	0	0	7	0	4	0	0
18	7	15	13	0	0	0	7	6	0	0	0	x	0	0	0	8	7	0	0	0
21	21	7	14	7	0	7	7	5	0	0	0	0	x	0	7	0	7	0	0	0
30	13	7	0	7	0	0	0	11	0	8	0	0	0	x	7	21	34	12	8	0
32	9	0	8	14	12	0	13	8	0	0	0	7	0	8	x	7	0	0	0	0
34	29	44	33	61	0	7	0	5	0	7	0	0	0	15	7	x	23	5	0	0
36	24	7	0	14	13	0	8	7	7	0	0	16	7	29	7	24	x	0	7	0
41	4	4	14	18	0	7	0	2	7	0	0	0	0	0	0	8	0	x	8	0
42	6	0	4	7	0	0	0	11	0	0	0	0	0	31	7	0	0	7	x	0
44	3	13	12	18	7	0	7	7	7	0	0	0	0	0	0	0	0	7	0	x
45	5	16	14	96	17	0	0	18	23	23	8	0	14	12	12	8	8	0	34	12
46	13	0	0	4	10	0	7	20	0	0	10	0	0	14	0	14	0	7	8	7
47	10	10	0	4	0	0	0	12	0	0	0	0	0	0	0	20	8	7	0	0
50	6	7	0	13	0	0	0	11	7	0	0	0	7	0	0	21	7	0	0	8
54	3	0	0	36	0	0	0	2	8	0	0	0	8	8	0	0	0	0	0	0
55	0	0	7	31	7	0	0	2	0	0	0	0	0	0	7	8	0	0	0	7
71	16	0	16	14	0	0	0	4	0	0	0	0	0	0	0	17	21	0	0	0
59	32	14	14	25	0	0	0	13	25	13	0	8	0	0	7	20	22	0	14	0
57	10	19	27	21	7	0	7	8	0	4	0	8	0	0	8	0	8	14	0	7
62	7	0	0	8	0	0	0	12	0	0	0	0	0	0	7	0	7	0	8	0
63	14	0	0	15	0	7	0	1	4	0	7	0	0	0	0	0	7	0	0	0
64	7	0	13	7	0	7	14	2	0	0	0	0	0	0	25	7	0	7	0	0

TABLE 14 (contd.)

Destinations

Origins	01	02	03	05	07	08	09	13	14	15	16	18	21	30	32	34	36	41	42	44
66	0	11	16	23	13	0	4	5	7	0	0	0	0	22	0	23	0	14	7	16
67	2	7	7	13	7	0	0	6	7	7	0	0	0	0	0	7	0	0	0	0
69	26	20	10	23	11	8	10	49	10	5	0	10	1	6	10	53	12	14	7	4
70	62	4	6	12	7	2	1	0	4	4	1	1	4	2	0	9	2	3	4	9

TABLE 14 (contd.)

Origins	Destinations																
	45	46	47	50	54	55	56	71	59	57	62	63	64	66	67	69	70
01	26	2	18	11	11	15	0	0	3	10	7	27	0	35	9	64	52
02	16	0	18	7	0	0	0	0	0	32	0	0	8	8	7	36	7
03	22	7	8	0	14	7	0	8	8	23	8	0	10	22	7	15	10
05	99	0	18	35	13	20	0	0	0	25	0	27	28	63	47	35	27
07	33	0	7	0	7	7	0	0	0	27	0	0	8	13	13	20	5
08	0	0	7	0	0	0	0	0	0	0	0	0	7	0	0	0	0
09	0	8	0	0	0	0	0	0	0	0	0	0	0	7	0	2	0
13	28	5	7	9	6	8	3	3	10	9	12	3	0	10	8	51	3
14	0	0	0	13	0	0	0	0	0	0	0	0	7	7	7	15	1
15	30	0	8	0	0	0	0	8	8	0	0	7	0	0	0	27	5
16	23	4	0	0	0	0	0	0	0	0	0	0	0	4	0	3	2
18	0	0	8	0	8	0	0	0	0	0	0	0	0	0	0	3	9
21	29	0	0	0	8	0	0	0	0	7	0	13	0	0	7	3	1
30	8	0	8	0	0	0	0	0	7	0	0	7	0	26	0	7	13
32	26	0	7	0	0	0	0	0	0	0	0	0	12	4	7	14	1
34	12	0	16	21	14	0	0	0	7	19	0	4	20	14	0	21	4
36	8	0	0	39	0	0	0	0	4	7	0	7	0	4	0	6	4
41	10	0	0	0	0	7	0	0	14	14	0	0	7	10	7	10	12
42	22	0	0	0	0	8	0	0	8	7	0	0	0	16	13	24	10
44	14	0	0	0	0	10	0	8	0	0	0	0	0	31	0	30	8
45	x	0	8	13	4	0	0	8	0	14	0	7	8	12	13	12	5
46	21	x	0	0	0	0	0	0	0	7	0	4	0	8	7	21	13
47	0	0	x	0	8	0	0	7	0	0	0	0	0	0	7	12	3
50	8	4	7	x	22	0	0	0	8	30	8	7	7	0	7	14	13
54	0	0	0	8	x	8	0	0	0	29	0	8	0	13	7	3	4
55	0	0	0	0	8	x	0	0	0	8	0	0	4	7	8	5	3
71	8	0	0	22	8	0	8	x	0	21	0	0	7	25	0	13	4
59	16	0	22	14	29	8	0	0	x	0	0	7	0	38	7	81	20
57	8	0	0	16	14	0	0	0	0	x	0	14	0	7	0	22	8
62	7	0	0	0	0	0	0	0	0	0	x	0	0	0	0	29	36
63	7	0	0	0	0	8	4	0	0	13	0	x	0	0	0	11	0
64	0	0	0	0	14	7	0	0	7	14	0	0	x	31	0	2	1
66	10	4	14	0	7	0	8	13	4	0	0	8	33	x	7	12	2

TABLE 14 (contd.)

Destinations

Origins		45	46	47	50	54	55	56	71	59	57	62	63	64	66	67	69	70
	67	29	7	8	0	7	4	0	4	0	7	8	0	0	8	x	14	6
	69	42	4	12	7	4	8	0	3	32	16	15	2	7	23	24	x	110
	70	14	2	3	4	6	2	0	3	8	16	10	3	0	8	13	103	x

ation concerning; the road geometry and condition (from the street inventory), the area of the city (C.B.D., intermediate, outlying), and the speed and delay studies. Operating speeds were taken, where available, from the work sheets of the speed and delay studies. Where information from this source was not available, the operating speeds were estimated.

Two sources of "true" volumes were used for comparison purposes. The manual counts taken for the turning movement studies (during the peak hour 4.30 to 5.30 p.m.) provided one source. The other source used was the peak hour flow map. This map was prepared from a variety of sources; traffic counters, turning movement counts, parking survey, etc. Not all links had available count information from either source. The turning movement counts only covered certain intersections. The flow map, because of its small scale, was not suitable for determining the flow on all links. Hence, only the figures printed on this map were used. Since both of these sources covered the peak hour and the trip table covers a two hour period, they had to be factored up to a two hour period. This was achieved by utilizing the long term volume count information. The counts were adjusted upward by a factor ranging from 1.67 to 1.8[illegible]. Generally, the lower figure was used on the collector streets and the higher figure on the arterial streets.

The assigned link volumes for both resistance functions are shown in Table 15. The straight cost resistance function is shown as Y_1; the product resistance function as Y_2. The solution was generated utilizing the I.B.M. 7040 computor at the University of Waterloo.

TABLE 15

LINK INPUTS and OUTPUTS

Input							Output		
Link No.	ND1	ND2	Length[a]	Free Speed	Oper. Speed	True Count[b]	True Count[c]	$Y_1^{(d)}$	$Y_2^{(e)}$
1	1	2	7.20	30	26	560	590	649	684
2	1	17	1.64	20	18			95	91
3	2	1	7.20	30	26		570	619	666
4	2	3	1.34	25	18		610	592	565
5	2	14	2.34	15	9			86	109
6	2	74	0.85	20	13		440	552	673
7	3	2	1.34	25	18	620	770	671	687
8	3	4	1.14	20	14	690	600	689	621
9	4	3	1.14	20	14			733	708
10	4	5	1.32	20	9	590	590	599	585
11	4	73	0.90	15	13			132	35
12	5	4	1.32	20	9	870	870	666	674
13	5	6	1.25	20	18		720	686	732
14	5	15	0.93	15	8			250	242
15	5	21	0.74	15	15			590	529
16	6	5	1.25	20	10	670	720	694	690
17	6	7	0.82	20	18			692	732
18	6	22	1.03	15	12			40	0
19	7	6	0.82	20	18			720	690
20	7	72	0.77	25	18	760	771	815	
21	8	9	1.66	30	28		690	663	725
22	8	16	1.63	15	15			59	44
23	8	25	1.02	15	15			156	102
24	8	72	0.71	25	12	690		672	671
25	9	8	1.66	30	28	730	780	657	644
26	9	10	1.30	30	26		590	574	645
27	9	32	2.20	15	12			44	32
28	10	9	1.30	30	26		670	627	636
29	10	11	2.54	30	26	610		575	645
30	10	33	2.20	15	14			11	0
31	11	10	2.54	30	26			629	636
32	11	12	3.20	30	28		590	527	570
33	11	44	3.26	30	25	340	400	282	523
34	12	11	3.20	30	28	730	690	613	681
35	12	13	3.35	30	28		490	335	352
36	12	45	2.81	20	18			238	264
37	13	12	3.35	30	28		540	450	518
38	13	46	2.85	20	18			21	38
39	14	2	2.34	15	9	200		121	142
40	14	15	2.90	15	13			59	38
41	15	5	0.93	15	8	180		270	249

TABLE 15 (contd.)

	Input						Output		
Link No.	ND1	ND2	Length[a]	Free Speed	Oper. Speed	True Count[b]	True Count[c]	Y_1[d]	Y_2[e]
42	15	14	2.90	15	13			121	98
43	15	16	2.80	15	15			24	16
44	16	8	1.63	15	15	40		55	55
45	16	15	2.80	15	15			33	10
46	17	1	1.64	20	18			48	32
47	17	18	7.10	25	20			51	37
48	17	47	2.76	20	20			98	116
49	18	17	7.10	25	20			76	47
50	18	19	2.48	20	15			77	106
51	18	34	1.77	20	18	490		500	598
52	18	74	0.59	20	13			478	569
53	19	18	2.48	20	15			76	52
54	19	20	0.91	15	10			17	2
55	19	35	1.93	20	18			209	139
56	19	73	0.60	15	15			78	35
57	20	19	0.91	15	10			63	21
58	20	21	0.88	25	16			517	507
59	21	5	0.74	15	15	530	630	537	559
60	21	22	1.30	10	10			0	0
61	21	27	1.73	25	16		570	597	557
62	21	73	1.35	15	12			54	0
63	22	6	1.03	15	10			20	0
64	22	21	1.30	10	10			1	0
65	22	23	0.82	10	10			0	0
66	22	28	1.24	15	10			40	0
67	23	7	1.03	15	10			38	2
68	23	22	0.82	10	10			2	0
69	23	24	0.77	15	10			0	0
70	24	23	0.77	15	10			12	0
71	24	25	0.71	15	10			17	7
72	24	30	1.24	15	15			74	22
73	25	8	1.02	15	15	100		153	66
74	25	24	0.71	15	10			15	0
75	25	31	1.15	20	15			170	109
76	26	20	0.95	25	16			564	526
77	26	27	0.83	15	15			9	0
78	27	26	0.83	15	15			12	0
79	27	28	0.71	15	15			137	17
80	27	37	1.03	20	10	490		494	540
81	28	22	1.24	15	10			19	0
82	28	27	0.71	15	15			37	0
83	28	29	0.80	15	15			78	17
84	28	38	1.08	15	15			100	0
85	29	23	1.24	15	10			28	2
86	29	28	0.80	15	15			33	0
87	29	30	0.77	15	15			125	35

TABLE 15 (contd.)

Input								Output	
Link No.	ND1	ND2	Length[a]	Free Speed	Oper. Speed	True Count[b]	True Count[c]	Y_1[d]	Y_2[e]
88	30	29	0.77	15	15			41	2
89	30	31	0.71	20	15			126	112
90	30	40	1.04	15	15			143	93
91	31	25	1.15	20	15			164	66
92	31	30	0.71	20	15			147	187
93	31	32	1.66	20	18			115	76
94	31	41	1.06	20	15	90		178	163
95	32	9	2.20	15	12			58	32
96	32	31	1.66	20	18			78	48
97	32	33	1.70	20	18			22	4
98	32	42	1.08	15	15			63	73
99	33	10	2.20	15	14			10	0
100	33	32	1.70	20	18			14	3
101	33	43	1.12	15	15			21	4
102	34	18	1.77	20	18			451	555
103	34	35	2.28	25	22		370	446	645
104	34	48	0.98	20	18			357	165
105	35	19	1.93	20	18			79	65
106	35	34	2.28	25	20	450	400	456	612
107	35	36	0.92	25	20	700		603	784
108	36	26	1.05	25	18			561	526
109	36	35	0.92	25	20			484	677
110	36	37	0.83	25	15	490	370	468	394
111	36	49	0.95	30	20			819	997
112	37	36	0.83	20	10	1020	870	1010	920
113	37	38	0.68	25	20		620	623	540
114	38	28	1.08	15	15			25	0
115	38	37	0.68	25	20	760	640	671	527
116	38	39	0.82	25	22			675	540
117	39	29	1.04	15	10			66	18
118	39	38	0.82	25	22			648	527
119	39	40	0.77	25	22			615	522
120	40	39	0.77	25	22			654	527
121	40	41	0.71	25	23	660		647	550
122	41	31	1.06	20	15			231	223
123	41	40	0.71	25	23			544	461
124	41	42	1.65	30	25			530	444
125	41	71	3.40	20	15	205	250	220	124
126	42	32	1.08	15	15			97	95
127	42	41	1.65	30	25	560		460	410
128	42	43	1.93	30	25		480	489	452
129	42	56	4.32	20	15		100	90	28
130	43	33	1.12	15	15			12	3
131	43	42	1.93	30	25		470	416	380
132	43	44	2.54	30	22	420		471	456
133	43	66	7.43	20	17			39	0
134	44	11	3.26	30	25	250	250	250	403

TABLE 15 (contd.)

Input								Output	
Link No.	ND1	ND2	Length[(a)]	Free Speed	Oper. Speed	True Count[(b)]	True Count[(c)]	Y_1[(d)]	Y_2[(e)]
135	44	43	2.54	30	25		380	406	383
136	44	45	3.96	20	15			343	300
137	44	67	7.81	30	28	490	490	467	661
138	45	12	2.81	20	18			209	209
139	45	44	3.96	20	15			329	261
140	45	46	3.37	20	15			26	9
141	46	13	2.85	20	18			60	128
142	46	45	3.37	20	15			135	67
143	47	17	2.76	20	20			25	46
144	47	48	7.25	20	18		100	73	52
145	48	34	0.98	20	18	390	390	372	228
146	48	47	7.25	20	18		160	106	88
147	48	49	3.10	20	18			97	25
148	48	53	2.82	20	20	300	340	306	143
149	49	36	0.95	30	20	840	840	745	916
150	49	48	3.10	20	18			82	34
151	49	50	1.70	30	24			844	1000
152	50	49	1.70	30	24			754	929
153	50	54	1.84	30	26		800	812	968
154	51	52	2.25	20	10			158	183
155	51	59	2.37	30	27			42	41
156	52	51	2.25	20	10			42	41
157	52	53	1.42	20	10	240		241	253
158	52	57	1.22	20	15			125	106
159	53	48	2.82	20	20	430	390	369	233
160	53	52	1.42	20	10			168	147
161	53	54	2.88	20	17		220	136	116
162	53	58	1.30	20	20			209	114
163	54	50	1.84	30	26		800	719	894
164	54	53	2.88	20	17	120	120	135	129
165	54	55	3.69	20	17			147	51
166	54	61	2.18	30	20			611	783
167	55	54	3.69	20	17			198	149
168	55	56	1.57	20	17			90	9
169	55	64	2.97	25	22		250	233	136
170	55	71	1.27	25	22			156	91
171	56	42	4.32	20	15			63	23
172	56	55	1.57	20	17			64	4
173	56	65	3.00	25	23	100	100	134	21
174	57	52	1.22	20	15			83	71
175	57	58	1.68	20	15			154	166
176	58	53	1.30	20	20	260		200	84
177	58	57	1.68	20	15			230	249
178	58	60	1.05	20	20	220		165	96

TABLE 15 (contd.)

Input								Output	
Link No.	ND1	ND2	Length[(a)]	Free Speed	Oper. Speed	True Count[(b)]	True Count[(c)]	Y_1[(d)]	Y_2[(e)]
179	59	51	2.37	30	27			158	183
180	59	60	4.13	20	20	240		291	266
181	60	58	1.05	20	20			231	149
182	60	59	4.13	20	20			86	87
183	60	61	1.74	20	17			221	286
184	60	68		20	20	280	200	180	77
185	61	54	2.18	30	20			534	690
186	61	60	1.74	20	17	100		161	196
187	61	62	1.38	30	27	710	700	645	832
188	62	61	1.38	30	27		600	509	649
189	62	63	3.50	30	24		290	338	430
190	62	68	1.38	25	23			481	500
191	63	62	3.50	30	24	270	300	233	313
192	63	64	1.95	30	27			249	331
193	64	55	2.97	25	22		250	185	101
194	64	63	1.95	30	27			201	272
195	64	65	1.56	30	17	390	400	335	348
196	65	56	3.00	25	23			104	34
197	65	64	1.56	30	17		400	248	262
198	65	66	2.39	30	24			442	354
199	66	43	7.43	20	17			21	0
200	66	65	2.39	30	24	350		325	281
201	66	67	3.15	30	24	260	260	161	208
202	67	44	7.81	30	28	380	270	315	432
203	67	66	3.15	30	24		300	197	307
204	67	70	4.72	40	32		340	297	359
205	68	60	2.80	20	20	180	200	102	41
206	68	62	1.38	25	23	480		396	380
207	68	69	2.10	20	20		700	632	576
208	69	68	2.10	20	20		470	468	421
209	69	70	15.10	58	55		270	234	413
210	70	67	4.72	40	32	210	220	249	302
211	70	69	15.10	58	55		300	212	400
212	71	41	3.40	20	15	190	200	220	120
213	71	55	1.27	25	22			295	244
214	72	7	0.77	25	18			657	668
215	72	8	0.71	25	24	620		699	790
216	72	24	1.03	15	10			87	29
217	73	4	0.90	15	10	210		108	35
218	73	19	0.60	15	15			159	35
219	73	74	2.50	15	12			2	0
220	74	2	0.85	20	13	400	400	477	569
221	74	18	0.59	20	13			550	673
222	74	73	2.50	15	12			6	0

TABLE 15 (contd.)

Notes: a) Length: 1" = 400 ft.

b) Peak Period Manual Counts 4:30 to 5:30 p.m. expanded to two hour counts

c) Peak Period Counts shown on flow map expanded to two hour counts

d) Resistance function R = K.M.S(p)

e) Resistance function R = K.M.s(p) . t(p)

Analysis of Results

Table 16 shows the comparison between the manual counts and the assigned volumes using the two postulated resistance functions. The average differences, over all volume classes, between the two postulated resistance functions were not significantly different. However, the variability (i.e. the variance) over all classes of the product resistance function was significantly greater than the variability of the straight cost function. To determine the statistical basis for the above statement an F test was used on the pooled variances of the differences, over volume of all classes, between the two postulated resistance functions. The Bartlett test (40) was first used to check for homogeneity of variances within each resistance function. At the 10 percent level of significance, the hypotheses that the variances within each class of resistance function were homogeneous were accepted. The hypothesis that there was no significant difference between the variances of the two resistance functions was rejected at the 1 percent level of significance. Based on this information, it may be concluded that for Brockville data the straight cost resistance function was a better predictor of link flows than the product resistance function. The average total error, including all of the sources previously mentioned, was less than 5 percent under both postulated resistance functions.

Table 17 shows the comparison between the counts obtained from the expanded figures on the flow chart and the assigned link volumes - using the two postulated functions. Again, using the Bartlett test (40) at the 10 percent level of significance the variances of the differences within each resistance function were homogeneous. The F

test showed, at the 1 percent level of significance, that the variability of the product resistance function was significantly greater than the variability of the straight cost function taken over all volume classes. Again, it may be concluded that the straight cost resistance function is a better predictor of link flows, for Brockville than the product function.

A comparison between the two "true" count sources also indicated, at the 5% level of significance, that there was no significant difference between the sources.

Based on these findings, it was concluded that the linear graph algorithm developed in this thesis is a good predictor of traffic flow.

TABLE 16

Comparison of Manual Counts with Assigned Volumes

Total Measured Volume	24,415	
Total Assigned Volume	23,345 (a)	23,493 (b)
Total Percent Error	-4.4 (a)	-3.8 (b)

Volume Group	No. Links	Ave. Count	Ave.(a) Assigned	Ave.(a) Diff.	Std.(a) Dev.	Ave.(b) Assigned	Ave.(b) Diff.	Std.(b) Dev.
0-99	3	57	98	+41	40.8	73	+16	56.7
100-199	8	149	166	+17	60.6	109	-40	93.8
200-299	12	237	202	-35	55.2	193	-44	112.9
300-399	6	358	323	-35	27.8	328	-30	132.9
400-499	9	460	454	- 6	50.3	494	+34	135.8
500-599	4	560	561	+ 1	77.5	560	0	113.5
600-699	7	651	664	+13	40.8	667	+16	92.0
700-799	6	732	660	-72	44.4	714	-18	132.1
800-899	2	855	706	-149	77.0	795	-60	192.0
>900	1	1020	1010	- 10	-	920	-100	-
Totals	58	508	484	-24	52.5	485	-26	118.8

a) Straight Cost Resistance Function R = K.M.S(p)

b) Product Resistance Function R = K.M.s(p). t(p)

TABLE 17

Comparison of Flow Map Volumes with Assigned Volumes

Total Measured Volume	30,790	
Total Assigned Volume	29,139[(a)]	29,684[(b)]
Total Percent Error	-5.4	-3.6

Volume Group	No. Links	Ave.(a) Count	Ave.(a) Assigned	Ave.(a) Diff.	Std. Dev.	Ave.(b) Assigned	Ave.(b) Diff.	Std. (b) Dev.
100-199	5	116	108	- 8	34.5	64	-52	36.2
200-299	13	242	217	-25	51.0	225	-17	128.2
300-399	11	351	343	- 8	71.0	351	0	135.1
400-499	11	440	411	-29	90.1	477	+37	137.2
500-599	7	577	574	- 3	56.2	604	+27	53.6
600-699	9	639	614	-25	60.1	611	-28	55.5
700-799	6	732	664	-68	37.6	693	-39	100.5
800-899	5	836	790	-46	47.9	874	+38	138.0
Totals	67	492	466	-26	62.5	487	- 5	112.2

a) Straight Cost Resistance Function R = K.M.S(p)

b) Product Cost Resistance Function R = K.M.s(p).t(p)

CONCLUSIONS AND RECOMMENDATIONS

Summary and Conclusions

1. A functional relationship (value function), based on psychological factors, that describes the aggregate of subjective values that travellers use in choosing a particular route could not be formulated at this time.

2. The value functions in terms of cost were formulated to reflect the indeterminate subjective values used by travellers. A value function based on a relationship between speed and cost, where the cost included time, operating, accident and quality of flow costs was a better predictor of these subjective values than the value function represented by the product of cost (exclusive of time) and time.

3. A path or route determination technique which utilizes the empirical evidence from diversion studies was formulated. It proved to be efficient and conceptually sound.

4. A modification of the techniques of linear graph theory was used to assign traffic amongst the various paths developed by the path determination algorithm. Using the postulated value functions, it was found that this model was a good predictor of link flows.

5. The advantages of this algorithm over the current assignment techniques are as follows:

a) The paths developed reflect empirical studies. More than one path between any origin-destination pair may be developed. Further, "demand" rather than "restraint" paths may be formulated.

b) The calculation of these paths are less time consuming than the current multiple path restraint techniques.

c) The linear graph technique allows demand and restraint assignment to alternate paths.

d) Fewer iterations are required for the restraint solution.

Recommendations for Further Study

The following items are recommended for further study:

1. A comparison of the results obtained by the proposed algorithm with cities other than the one chosen in this study.
2. A psychological investigation and micro-field studies into the factors and behaviour of travellers to formulate a more deterministic value function.
3. An investigation of the proposed technique into assignments that involve modal splits.
4. An investigation into the possibilities of combining a trip distribution and assignment model by systems techniques.

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A P P E N D I X

APPENDIX A

Definitions

Node	The point of intersection between two segments of a route
Link	The segment of a route determined by two nodes
Route or Path	A series of connected links between the centroids of two zones
Zone	A subarea of the study area
Centroid	A point in a zone at which all trips are assumed to originate or terminate
Modal Split	The proportioning of trips between private and transit vehicles
Demand or Unrestricted Flow	The number of trips that have been allocated to a link or route under some given or assumed condition. Demand flow can be expressed in the number of vehicles per unit time without knowledge of the capacity of the links involved.
Minimum Path Tree	A series of connected links from an origin to a destination such that no circuits are formed and which minimizes some travel function.
Capacity Restraint or Demand Restraint	A functional relationship between the demand for a particular facility and the travel time on that facility. Demand restraint would be a better term since on any link there is maximum flow that it may accommodate, but greater demand may exist for the facility. The functional relationship then reflects queueing time as well as moving time.
Diversion Assignment	The proportioning of trips between <u>two</u> zones to <u>two</u> routes on the basis of some type of diversion curve.
All-or-Nothing Assignment	The allocation of all interzonal transfers for a zonal pair to the optimized route.

Trip Table	A table showing the number of trips between all origins and destinations in a study area.
System	A set of components interconnected in some orderly manner with relationships between the components and their attributes (the properties of the component)
Environment	For a given system, the environment is a set of all components or objects outside the system whose attributes are changed by the system or a change in whose attributes affect the system.
Two Terminal Component	A component that is connected to other components at exactly two points, areas or regions in the construction of a system.
Oriented Element	An oriented line segment together with its distinct ends
Vertex	An end point of an element
Oriented Linear Graph	A set of oriented elements, no two of which have a point in common that is not a vertex
Subgraph	A subset of the elements of a graph
Incident	A vertex and an element are incident with each other if the vertex is an end point of the element
Circuit	A circuit is a closed path, where the vertices have two and only two elements incident thereto.
Tree	A tree is a connected subgraph of a graph such that it contains all the vertices of the graph but no circuits
Branch	An element of the tree is a branch
Complement (Cotree)	The complement of a subgraph is the set of elements of the graph not contained in the subgraph
Chord	An element of the complement of a tree
Cut Set	A cut set is a set of elements in a graph such that: 1) the removal from the graph of these elements reduces the rank of the graph by one; and 2) no proper subset of the cutset has property 1)
Free Speed	The maximum speed selected by an operator on a particular route section at extremely low densities
Mean Free Speed	The average of the distribution of free speeds. These speeds usually approach the speed limit of thelink.

APPENDIX B

List of Fortran Definitions

ACT (I) - the operating speed on link I (actual speed from study)

ALPTHS - the path finding routine

AV - a constant in the delay function

CAP - the capacity of a link (v.p.h. or v.p.d.)

CONST - the diversion factor

COUNT - the number of iterations in the linear graph routine

D(I) - the length of a link in miles

DELAY(I) - the delay time of the link

DMIN(I,J) - the minimum path resistance from I to J

FL(I) - the flow on link I

FS(I) - the free speed on link I

FV - a constant in the delay function

JKOUNT also (MKOUNT) - a space to store the number of paths found from an origin to a destination

J(I) - the number of the destination node

KLINK - the next link in the link table after LINK

KOUNT - (the number of nodes on a path) + 1

KPTH(I,J) - a matrix to store the diversion paths between origin I and destination J

KTAPE - an index of 1 if the paths have already been found

KTIME - an index of 1 for hourly flows; 2 for daily flows

KTYPE (also NTYPE) - the type of link, 1 = arterial; 2 = free-way

LNKIND - a link indicator

LINK - the link being checked to be added to a path if suitable

LKST(I) - the number of the first link whose beginning node is node I. The links must be arranged in the link table in ascending order (see Table 15)

MPTHS - the R.R.L. minimum path algorithm

N1 - the origin node

N2 - the destination node

N(I) - the number of the origin node in the R.R.L. algorithm

NCUM - node number entry in cumulative table in R.R.L. algorithm

NDIND - an indicator NDIND(I) = 1 if I is not on the path; NDIND(I) = 2 if node I on the path; NDIND(N2) = 3.

NEND - the terminal node of link LINK in the diversion path routine

NFIN - the destination of the minimum path in the R.R.L. algorithm

NHOME - the origin of the minimum path in the R.R.L. algorithm

NLINKS - the number of links in a network

NLOADS - the number of origins in a network

NN - a vector to store the number of the links on a path

NNODES - the number of nodes in a network

NODE - the beginning node of a link

NPT - the number of paths allowed between an origin and destination

NPTH - the number of paths available

NRECVS - the number of destination nodes

NTRIPS(I,J) - the traffic flow from I to J

NUM - the number of paths to be considered

NZONE - the number of nodes in the R.R.L. algorithm

R(I) - the resistance of link I

RM(I,J) - the maximum allowable path resistance from I to J

SPTH - a vector to store the number of nodes on a path

SR - a vector to store the average values of the flows

SY(I) - a vector to store the most recently calculated flow values

TCUM - the cumulative time to a point from an origin in the R.R.L. algorithm

TFL(I) - the path flow

TLINK(I) - the resistance of link I in the R.R.L. algorithm

TMIN - a variable used for searches for minimum entry in R.R.L. algorithm

TRIPS - the trip table

TR(I) - the total path resistance on the I'th path N1 to N2

TSUM - a vector to store the cumulative resistance in the diversion path routine

TSUMM - the cumulative resistance along a path

X - the resistance along a path if a link is added to a path in the diversion routine

APPENDIX C

Computor Programs for Linear Graph Assignment Algorithm

(Fortran IV Coding for I.B.M.7040)

APPENDIX C-1

Minimum Path Programme

```
      SUBROUTINE MPATHS (NHOME, NFIN, NN, TSUM)
      DIMENSION TLINK (300), N(300), J(300)
      DIMENSION NN(80), TSUM (80), TCUM (80), NCUM(80)
      COMMON NZONE, NLINK, N, J, TLINK
62    DO2I = 1,NZONE
 2    TSUM(I) = 9999.99
      NTREE = 1
      NN(NHOME) = 0
      TSUM(NHOME) = 0
      NCM = 0
      NM = NHOME
      IF(NHOME-NFIN)6,50,6
 6    DO7I = NM, NLINK
      IF (N(I)-NM) 7,3,8
 3    K = J(I)
      IF(TSUM(K)-9999.99)7,18,7
18    NCM = NCM+1
      TCUM(NCM) = TSUM(NM) + TLINK(I)
      NCUM(NCM) = I
 7    CONTINUE
 8    TMIN = 9999.99
```

```
   DO9K = 1, NCM
   IF (TMIN-TCUM(K)) 9,9,10
10 TMIN = TCUM(K)
   L = NCUM(K)
   M = K
 9 CONTINUE
   K = J(L)
   IF (TSUM(K) - TMIN) 11,11,13
11 I = 1
   GO TO 12
13 TSUM(K) = TMIN
   NN(K) = L
   IF (K- NFIN)1,50,1
 1 I = 0
   NTREE = NTREE + 1
   IF (NTREE - NZONE) 12,50,12
12 DO14NM = M, NCM
   TCUM(NM) = TCUM(NM + 1)
   NCUM(NM) = NCUM(NM + 1)
   IF(NM + 1 - NCM)14,15,14
14 CONTINUE
15 NCM = NCM - 1
   IF (I)8,17,8
17 NM = K
   GO TO 6
50 RETURN
   END
```

APPENDIX C-2

Path Determination Programme

```
      SUBROUTINE ALPTHS (N1,N2, NPT,RM,JKOUNT,KPTH,TR,DMIN)
      DIMENSION NDIND(80),TSUM(80),NN(80),KPTH(80,15)
      DIMENSION ND1(300),ND2(300),R(300), LKST(80)
      DIMENSION TR(15), DMIN(80,80)
      COMMON NNODES,NLINKS, ND1,ND2,R,LKST
      INTEGER SPTH
  511 FORMAT(10H MORE THAN,13,22H PATHS HAVE BEEN FOUND
      15H FROM,I3,3H TOI3)
      TMAX = 0.0
      TSUMM = 0.0
      MKOUNT = 0
      NODE = N1
      LINK = LKST(NODE)
      DO77I = 1,NNODES
   77 NDIND(I) = 1
      NDIND(N1) = 2
      NDIND(N2) = 3
      KOUNT = 2
      TSUM(1) = 0.0
   71 NEND = ND2(LINK)
      IND = NDIND(NEND)
```

```
      GO TO (72,73,72),IND
   72 X = TSUMM + R(LINK)
      IF (X + DMIN(NEND,N2).GE.RM)GO TO 73
      IF (IND.NE.3.AND.LKST(NEND).EQ.0) GO TO 73
      TSUM(KOUNT) = X
      TSUMM = X
      NN(KOUNT) = LINK
      GO TO (75,73,76),IND
   75 NDIND(NEND) = 2
      LINK = LKST(NEND)
      KOUNT = KOUNT + 1
      NODE = NEND
      GO TO 71
   76 MKOUNT = MKOUNT + 1
      IF (MKOUNT.EQ.NPT + 1) PRINT 511,NPT, N1, N2
      IF (MKOUNT.LE.NPT) GO TO 2
      IF (TSUMM.GE.TMAX) GO TO 73
      JKOUNT = KMAX
      GO TO 1
    2 JKOUNT = MKOUNT
      IF (TSUMM.LE.TMAX) GO TO 1
      TMAX = TSUMM
      KMAX = MKOUNT
    1 K1 = KOUNT + 2
      KPTH(1,JKOUNT) = KOUNT
      TR(JKOUNT) = TSUMM
      DO78I = 2,KOUNT
```

```
      K2 = K1 - I
   78 KPTH(I, JKOUNT) = NN(K2)
      IF (MKOUNT.LE.NPT) GO TO 73
      TMAX = 0.0
      DO3I = 1,NPT
      IF (TR(I).LE.TMAX) GO TO 3
      KMAX = I
      TMAX = TR(I)
    3 CONTINUE
   73 KLINK = LINK + 1
      IF (ND1(KLINK).NE.NODE) GO TO 74
      LINK = KLINK
      GO TO 71
   74 IF (NODE.NE.N1)NDIND (NODE) = 1
      KOUNT = KOUNT - 1
      IF (KOUNT.EQ.1) GO TO 4
      LINK = NN(KOUNT)
      NODE = ND1(LINK)
      TSUMM = TSUM(KOUNT - 1)
      GO TO 73
    4 IF (MKOUNT.GT.NPT) JKOUNT = NPT
      RETURN
      END
```

APPENDIX C-3

Assignment Programme

```
      DIMENSION SY(300), NN(80),TSUM(80),SPTH(80),LKST(80),DELAY(300)

      DIMENSION CAP(300),R(300),D(300),FS(300),FL(300),ND1(300),
      ND2(300)

    1 KTYPE(300),LNKIND(300),TR(15),TFL(15),FV(2),AV(2),TABLE(30),
      NTRIP

    1S(80,80),KPTH(80,15),NTYPE(80),DMIN(80,80),SR(300)

      INTEGER SPTH

      COMMON NNODES,NLINKS,ND1,ND2,R,LKST,KTYPE,CAP,D,FS,FL,KTIME,
      FV,AV,1TABLE,AK,DELAY

      EQUIVALANCE (DMIN(1,1),SR(1))

  401 FORMAT (3I4,5F10.4)

  403 FORMAT (F12.8)

  404 FORMAT (1X,4I4,3F10.2,F12.6)

  409 FORMAT (20I4)

  502 FORMAT (1X,2I3,F14.8,27I3,2(/21X,27I3))

  503 FORMAT (20HOMIN. PATH TREE FROM,I3/)

  506 FORMAT (6HOERROR,I3)

  507 FORMAT (1X,I3,I5,10F12.6/(9X,10F12.6))

  508 FORMAT (10HO  O. D./27H NODE NODE TRAFFIC ON PATHS)

  509 FORMAT (14HOO-NODE D-NODE,6X,10HRESISTANCE)

  515 FORMAT (11HOLINK FLOWS/14HOO-NODE D-NODE, 6X,4HFLOW)

  516 FORMAT (21H1LINEAR GRAPH ROUTINE/),
```

```
 520  FORMAT (1X, I5,I7,F15.8)
 530  FORMAT (10HOITERATION,F10.2)
 535  FORMAT(59H1 LINK ND1 ND2 TYPE CAPACITY LENGTH SPEED RESISTANCE
 564  FORMAT (16H1ALL KNOWN PATHS)
C***  DEFINE TAPE UNITS
      KUNIT = 0
      LSU = 1
      REWIND KUNIT
      REWIND LSU
C***  INITIALIZE PARAMETERS FOR RESISTANCE AND PATH-FINDING ROUTINES
      READ401,KTIME,NPT,KTAPE,CONST
      FV(1) = 1.2
      FV(2) = 1.98
      AV(1) = EXP(7.5)
      AV(2) = EXP(4.54)
      DO201I = 2,29
 201  READ403,TABLE(I)
      AK = 1.0
C***  READ NUMBERS OF NODES,LINKS,ETC., AND TRIP TABLE FROM CARDS
      READ 409,NNODES,NLINKS,NLOADS, NRECVS
      DO405I = 1,NNODES
      NTYPE(I) = 0
      DO405J = 1,NNODES
 405  NTRIPS(I,J) = 0
      READ409,(KTYPE(I), I=1,NLOADS)
      DO410I = 1,NLOADS
```

```
      K1 = KTYPE(I)
  410 NTYPE(K1) = 1
      READ409,(KTYPE(I),I = 1,NRECVS)
      DO407I = 1,NNODES
      IF(NTYPE(I).EQ.0)GO TO 407
      READ409,(LNKIND(K),K=1,NRECVS)
      DO406K = 1,NRECVS
      K1 = KTYPE(K)
  406 NTRIPS(I,K1) = LNKIND(K)
  407 CONTINUE
C*** READ AND PRINT LINK DATA
      PRINT 535
      DO603I = 1,NLINKS
    1 READ401,ND1(I), ND2(I),KTYPE(I),CAP(I),D(I),FS(I),DELAY(I),ACT
      FL(I) = 0.0
  102 R(I) = RES(I)
  603 PRINT404,I,ND1(I),ND2(I),KTYPE(I),CAP(I),D(I),FS(I),R(I)
C*** IF KTAPE EQUALS L,SKIP THE PATH-FINDING ROUTINE
      IF (KTAPE.EQ.1)GO TO 53
C*** SET LKST(I) EQUAL TO THE NUMBER OF THE FIRST LINK FROM NODE I
      DO79I = 1,NNODES
   79 LKST(I) = 0
      NODE = ND1(1)
      LKST(NODE) = 1
      DO83I = 1,NLINKS
      IF (ND1(I).EQ.NODE) GO TO 83
      NODE = ND1(I)
```

```
      LKST(NODE) = I
   83 CONTINUE
      DO1300N1 = 1,NNODES
C***  FIND THE MINIMUM PATH TREE FOR EACH NODE, N1
      CALL MPATHS(N1,0,NN,TSUM)
      DO2J = 1,NNODES
    2 DMIN(N1,J) = TSUM(J)
      IF (NTYPE(N1).EQ.0)GO TO 1300
      PRINT503,N1
      DO13N2 = 1,NNODES
      IF ((N1.EQ.N2).OR.(NTRIPS(N1,N2).EQ.0)) GO TO 13
      IND = 0
C***  SET KPTH(I) EQUAL TO THE NUMBER OF THE ITH LAST LINK AND SPTH
      (I+1) EQUAL TO THE ITH NODE IN THE MINIMUM PATH FOR EACH O-D
      PAIR (N1,N2)
    5 NUM = 2
      NODE = N2
      SPTH(80) = N2
    6 LNK= NN(NODE)
      KPTH(NUM,1) = LNK
      NUM = NUM + 1
      NODE = ND1(LNK)
      LNK = 82-NUM
      SPTH(LNK) = NODE
      IF (NODE-N1)6,7,6
    7 KPTH(1,1) = NUM - 1
      KU = 82 - NUM
```

```
C*** PRINT MINIMUM PATH FROM N1 TO N2 FOR EACH O-D PAIR (N1,N2)
      PRINT502,N1,N2,TSUM(N2),(SPTH(I),I = KU,80)
      NPN1N2 = 1
C*** WRITE MINIMUM PATH FROM N1 TO N2 ON TAPE FOR EACH O-D PAIR
     (N1,N2)
 12   WRITE (LSU) N1,N2,NPN1N2
      WRITE (LSU) (KPTH(I,1),I = 1, NNODES)
 13   CONTINUE
 1300 CONTINUE
C*** SWITCH TAPE UNITS
      CALL STAPES (KUNIT,LSU)
      PRINT564
 22   DO2400N1 = 1,NNODES
      IF (NTYPE(N1).EQ.0) GO TO 2400
      DO24N2 = 1,NNODES
      IF (N1.EQ.N2.OR.NTRIPS(N1,N2).EQ.0)GO TO 24
C*** READ THE N1/N2 DATA FROM TAPE FOR EACH O-D PAIR (N1,N2)
 23   READ (KUNIT) NB,NL,NUM
      KERR = 4
      IF ( I ABS(NB-N1) + IABS(NL - N2).GT.0)GO TO 99
      READ (KUNIT) ((KPTH(I,J), I = 1,NNODES), J = 1,NUM)
C*** SET RM EQUAL TO CONST TIMES THE RESISTANCE OF THE MINIMUM PATH
     FROM N1 TO N2
      RM = CONST*DMIN(N1,N2)
      KERR = 12
      IF (LKST(N1).EQ.0) GO TO 99
C*** FIND AND PRINT ALL PATHS FROM N1 TO N2 FOR WHICH THE RESISTANCE
     IS LESS THAN RM
```

```
      CALL ALPTHS (N1,N2,NPT,RM,NUM,KPTH,TR,DMIN)

      KERR = 17

      IF (NUM.EQ.0) GO TO 99

      DO9I = 1,NUM

      SPTH(80) = N2

      KLM = KPTH (1,I)

      DO8J = 2,KLM

      LNK = KPTH(J,I)

      NODE = ND1(LNK)

      KRT = 81 -J

    8 SPTH(KRT) = NODE

      KLM = 81 - KLM

    9 PRINT502,N1,N2,TR(I),(SPTH(J), J = KLM,80)

C***  WRITE N1/N2 DATA ON TAPE

      WRITE (LSU) N1,N2,NUM

      WRITE (LSU) ((KPTH(I,J), I = 1,NNODES), J = 1,NUM)

   24 CONTINUE

 2400 CONTINUE
C***  SWITCH TAPE UNITS

      CALL STAPES(KUNIT,LSU)

C***  SET LINK INDICATOR LNKIND EQUAL TO 1 FOR ALL LINKS, AND
      SET SY(I) EQUAL TO THE FLOW ON LINK  I

   53 PRINT516

      DO35I = 1,NLINKS

      SR(I) = 0

      LNKIND(I) = 1

   35 SY(I) = FL(I)

C***  SET COUNT EQUAL TO ZERO
```

```
      COUNT = 0.0
C*** LINEAR GRAPH ROUTINE
C*** INCREASE COUNT BY 1
   36 COUNT = COUNT + 1.0
      PRINT530,COUNT
      PRINT508
C*** SET FLOWS ON LINKS EQUAL TO ZERO
      DO37I = 1,NLINKS
   37 FL(I) = 0.0
      DO3900N1 = 1,NNODES
      IF (NTYPE(N1).EQ.0) GO TO 3900
      DO39N2 = 1,NNODES
      IF(N1.EQ.N2.OR.NTRIPS(N1,N2).EQ.0) GO TO 39
      KERR = 6
C*** READ DATA FROM TAPE FOR EACH O-D PAIR (N1,N2)
      READ (KUNIT) NB,NL,NUM
      IF(IABS(NB-N1)+ IABS(NL-N2).GT.0) GO TO 99
      READ (KUNIT) ((KPTH(I,J), I = 1,NNODES), J = 1,NUM)
      TRIPS = NTRIPS(N1,N2)
C*** SET TR(I) EQUAL TO THE TOTAL RESISTANCE ON THE ITH PATH FROM
      N1 TO N2
      DO38J = 1,NUM
      TR(J) = 0.0
      NL = KPTH(1,J)
      DO38I = 2,NL
      K = KPTH(I,J)
   38 TR(J) = TR(J) + R(K)
```

```
C***  USE THE LINEAR GRAPH SUBROUTINE TO SET TFL(I) EQUAL TO THE
      TRAFFIC TO BE ASSIGNED TO THE ITH PATH FROM N1 TO N2

      CALL LNGRPH(NUM,TRIPS,TR,TFL)

C***  PRINT THE PATH FLOWS

      PRINT507,N1,N2,(TFL(I), I = 1,NUM)

C***  INCREASE THE FLOW ON EACH LINK OF THE ITH PATH BY TFL(I)

      CALL ASSIGN(NUM,TFL,KPTH,FL)

      IF(COUNT.GT.1.0) GO TO 39

C***  WRITE THE N1/N2 DATA ON TAPE

      WRITE (LSU) N1,N2,NUM

      WRITE (LSU) ((KPTH(I,J),I = 1, NNODES), J = 1,NUM)

   39 CONTINUE

 3900 CONTINUE

C***  SWITCH TAPE UNITS

      CALL STAPES (KUNIT,LSU)

C***  PRINT LINK FLOWS

      PRINT515

      DO1002I = 1,NLINKS

 1002 PRINT520,ND1(I),ND2(I),FL(I)

C***  COMPARE THE FLOW ON EACH LINK WITH THE PREVIOUS FLOW STORED
      IN SY

      IND = 0

      DO40K = 1,NLINKS

      A = SR(K)

      B = FL(K)

      SIG = 0.1*A

      IF (ABS(A-B).LE.SIG) GO TO 40
```

```
C*** IF NEW FLOW IS SIGNIFICANTLY DIFFERENT,STORE NEW VALUE
     IN SY, AND SET FLOW EQUAL TO THE AVERAGE OF ALL FLOWS
     FOUND, SET LINK INDICATOR LNKIND EQUAL TO 1

     IND = 1

     C = COUNT

     IF (COUNT.EQ.1.0)C = 0.0

     SY(K) = FL(K)

     FL(K) = (C*A + B)/(C + 1.0)

     SR(K) = FL(K)

     LNKIND(K) = 1

  40 CONTINUE

C*** IF NO LINK FLOW HAS CHANGED SIGNIFICANTLY, GO TO THE FINAL-
     PRINTOUT ROUTINE

     IF(IND.EQ.O.AND.COUNT.NE.1.0) GO TO 41

C*** CALCULATE NEW RESISTANCE VALUES FOR LINKS ON WHICH THE FLOWS
     HAVE CHANGED

     DO42I = 1,NLINKS

     IF(LNKIND(I).EQ.1)R(I) = RES(I)

  42 LNKIND(I) = 0

C*** GO TO THE START OF THE LINEAR GRAPH ROUTINE

     GO TO 36

  99 PRINT506,KERR

     GO TO 62

C*** FINAL PRINTOUT ROUTINE

C*** PRINT THE FINAL LINK RESISTANCES

  41 PRINT509

     DO303I = 1,NLINKS

 303 PRINT520,ND1(I), ND2(I), R(I)
  62 CALL EXIT
     END
```

VITA

VITA

Wallace Alvin McLaughlin was born May 5, 1927, in Calgary, Alberta. He received his primary and secondary education in Saskatoon, Saskatchewan, and was graduated from Bedford Road Collegiate in 1945.

He received a Bachelor of Science in Civil Engineering degree from the University of Saskatchewan in 1951, and a Master of Science in Civil Engineering degree from Purdue in 1958.

From 1951 to 1961 he was employed by the Saskatchewan Department of Highways in various positions; from 1951 to 1955 as a Project Engineer, 1955 to 1957 as a Division Engineer, 1958 to 1961 as Senior Traffic Engineer.

In 1961 he was appointed to the staff of the University of Waterloo as Assistant Professor in the Department of Civil Engineering. He took a leave of absence to undertake additional graduate study at Purdue University. In 1964 he was promoted to the rank of Associate Professor at the University of Waterloo. He is a Canadian citizen.

He is a registered Professional Engineer in the Province of Ontario, a member of the Engineering Institute of Canada, and an associate member of the Institute of Traffic Engineers.